Advanced Finance and Stochastics

Book of Abstracts

Moscow, 24–28 June 2013

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Preface

International Conference “Advanced Finance and Stochastics” was held on June 24–28, 2013 at Steklov Mathematical Institute in Moscow, Russia.

This book contains abstracts of the 51 papers presented at the Conference, which cover areas such as stochastic control in finance, derivatives pricing and hedging, portfolio selection, statistics of financial data, risk theory and others.

The Conference was organized by Steklov Mathematical Institute, Laboratory for Structural Methods of Data Analysis in Predictive Modeling, Center for Structural Data Analysis and Optimization, and Institute for Information Transmission Problems. Financial support was provided by the Government of the Russian Federation, grant ag.11.G34.31.0073.

We dedicate the hosting of this event to the tercentenary of Jacob Bernoulli’s remarkable paper “Ars Conjectandi”, 1713.

The general co-chairmen

Albert Shiryaev

Vladimir Spokoiny

Scientific Committee:

D. Belomestny, M. Schweizer, A. Shiryaev, V. Spokoiny

Organizing Committee:

A. Shiryaev, V. Spokoiny, A. Kuleshov, N. Beyer, E. Burnaev, B. Kashin, A. Muravlev, A. Sergeev, T. Tolozova, P. Yaskov, M. Zhitlukhin, S. Zhulenev

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Conference program

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11:30 – 11:50	D. Rokhlin: *On a generalized shadow price process in utility maximization problems under transaction costs*
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Plenary talks

Optimal stopping via multilevel Monte Carlo

Denis Belomestny

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In this talk we present a general methodology towards solving high-deminsional optimal stopping problems via Multilevel Monte Carlo. The multilevel versions of the well known primal-dual, stochastic mesh and policy improvement algorithms will be introduced. We conduct a thorough complexity analysis of the proposed algorithms and illustrate their efficiency for several high-dimensional option pricing problem in finance.

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Developing long term yield curve models for low rate regimes

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After a brief discussion of alternative approaches to long term yield curve modelling, this talk will first discuss typical applications of these models to structured product pricing, investment and asset liability management. It will go on to evaluate existing models and their drawbacks before developing and testing a nonlinear 3 factor model appropriate for today's low rate environments which is based on a posthumously published suggestion of Fisher Black. The talk will conclude with a description of current research into computationally intensive calibration techniques for this model and a more complex model requiring the EM algorithm which involves iterative Kalman filtering and maximum likelihood parameter estimation. Progress to date and remaining challenges will be described.

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Two price valuation in continuous time

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Based on joint work with Dilip Madan, Martijn Pistorius, Wim Schoutens, Marc Yor.

In classical economic theory the law of one price prevails and market participants trade freely in both directions at the same price. This approach is appropriate for highly liquid markets such as e.g. stock exchanges. In the absence of perfect liquidity the law of one price should be replaced by a two price valuation theory where market participants continue to trade freely with the market but the terms of trade now depend on the direction of the trade. Typical examples of markets with poor liquidity are over the counter (OTC) markets or some markets for corporate bonds.

We develop here a static as well as a continuous time theory for two price economies. The two prices are termed bid and ask or lower and upper price but they should not be confused with the vast literature where bid-ask spreads are related to transaction costs or other frictions involved in modeling financial markets. The two prices arise on account of an exposure to residual risk that results from the absence of sufficient liquidity and cannot be eliminated. The prices are designed to make this exposure acceptable. Acceptability as a strict mathematical term is modeled by requiring positive expectations under a whole host $\mathcal{M}$ of test or scenario probabilities Q as described for example in Artzner, Delbaen, Eber and Heath (1999). As a consequence the bid or lower price of a cash flow which is represented by a random variable X is given by the infimum of test valuations

$$b(X) = \inf_{Q\in\mathcal{M}} E^Q[X]$$

while the ask or upper price turns out to be the supremum of the same set of test valuations

$$a(X) = \sup_{Q\in\mathcal{M}} E^Q[X].$$

The resulting pricing operators are nonlinear operators on the space of random variables, with the lower price being concave and the upper price convex. In particular the upper price of a portfolio of financial instruments (risks) is smaller than the sum of the prices of the components of the portfolio while the lower price is similarly higher. Under the appropriate assumptions the computation of these nonlinear expectations can be operationalized. The lower price can be expressed as an expectation which is computed after distorting the risk distribution function by composing it with a concave distribution function Ψ on the unit interval. The price is then represented in the form

$$b(X) = \int_{-\infty}^{\infty} x d\Psi(F_X(x)).$$

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A suitable family of distortions which can be calibrated to market price data is given by the minmaxvar functions

$$\Psi^{\gamma}(x) = 1 - \left(1 - x^{\frac{1}{1+\gamma}}\right)^{1+\gamma} \qquad (0 \le x \le 1, \gamma \ge 0).$$

This static theory can be extended to a dynamically consistent nonlinear pricing approach in continuous time. For the case where the driving process is a diffusion process much progress in this direction has been made by the construction of $\mathcal{G}$-expectations by Peng (2007). We develop here a theory where the underlying uncertainty is given by a pure jump Lévy process $X = (X_t)_{t\ge 0}$ such as a hyperbolic, a variance gamma or a CGMY process. In this case the process is completely specified by a drift term α and a Lévy measure ν given via its density in the form $k(y)dy$. The corresponding infinitesimal operator $\mathcal{L}$ of the process is

$$\mathcal{L}u(x) = \alpha \frac{\partial u}{\partial x}(x) + \int_{\mathbb{R}} \left(u(x+y) - u(x) - \frac{\partial u}{\partial x}(x) y \right) k(y)dy.$$

If we denote by $u(x,t)$ the (risk-neutral) time zero financial value, when $X(0) = x$, of a claim paying $\phi(X_t)$ at time t, this function is the solution of a partial integro-differential equation (PIDE)

$$u_t = \mathcal{L}(u) - ru$$

subject to the boundary condition $u(x,0) = \phi(x)$ where r denotes a constant interest rate. This solution can also be expressed in the form

$$u(x,t) = E[e^{-rt}\phi(X_t) \mid X_0 = x].$$

Using integrability properties of the Lévy density the infinite Lévy measure is first transformed into a probability and then the integral term in the equation for the operator $\mathcal{L}$ is distorted similar to the distortion of the expectation in the static case. The bid price arises as the solution of the PIDE where the distorted operator is used. One gets the ask price as the negative of the bid price of the negative cash flow. To demonstrate that this general approach can be implemented we derive bid and ask prices for portfolios of derivatives as well as for perpetuities and insurance loss processes.

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Market-triggered changes in capital structure: equilibrium price dynamics

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Contingent capital is debt issued by a bank that converts to equity when the bank nears financial distress. This type of security has been proposed as a potential solution to the problem of banks that are "too big too fail," providing a private sector alternative to costly government bail-outs.

The biggest challenge to implementation is the selection of the trigger for conversion from debt to equity. Some have proposed triggers based on market prices, such as a decline in the bank's own stock price. However, using the market price of a firm's equity to trigger a change in the firm's capital structure creates a question of internal consistency because the value of the equity itself depends on the firm's capital structure.

We analyze the general problem of existence and uniqueness of equilibrium values for a firm's liabilities in this context, meaning values consistent with a market-price trigger. The liquidity of the triggering security, as measured by trading frequency, has important implications for this problem. In a static or discrete-time formulation of the problem, multiple solutions are possible. In contrast, we show that the possibility of multiple equilibria can largely be ruled out in continuous time. Continuous-time trading allows prices to fully adjust in anticipation of reaching a trigger.

Put abstractly, the problem we consider is the following. Given two martingales U and V on a time interval $[0, T]$, when does there exist a (unique) third martingale S, such that S coincides with U the first time S crosses a specified barrier, and $S_T = V_T$ if S never crosses the barrier. We interpret U and V as the prices of claims on post-conversion and no-conversion variants of a firm, and we seek an equilibrium price process S in which conversion is triggered by a barrier crossing by S itself. (This is analogous to pricing a "self-referential" barrier option in which the barrier is applied to the price of the barrier option itself, rather than to the underlying asset.) We impose conditions on U and V that lead to a unique solution and interpret these conditions in terms of contract features.

Within our general framework, existence of an equilibrium is ensured through appropriate positioning of the trigger level; in the case of contingent capital with a stock price trigger, we need the trigger to be sufficiently high. More generally, if the conversion is to be triggered by a decline in the market price of a claim, then the key condition we need is that the no-conversion price be higher than the post-conversion price when either is above the trigger. Put differently, we require that the trigger be sufficiently high to ensure that this holds. For the design of contingent capital with a stock price trigger, this condition may be interpreted to mean that conversion should be disadvantageous to shareholders. Our results apply as well to other types of changes in capital structure and triggers based on debt values as well as equity values.

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Approximation of nondivergent type parabolic PDEs in finance

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We study the spatial discretisation of the Cauchy problem

$$\frac{\partial u}{\partial t} = Lu + f \quad \text{in} \quad [0,T] \times \mathbb{R}^d, \quad u(0,x) = g(x) \quad \text{in} \quad \mathbb{R}^d, \tag{1}$$

where L is the second-order partial differential operator in the nondivergence form

$$L(t,x) = a^{ij}(t,x)\frac{\partial^2}{\partial x^i \partial x^j} + b^i(t,x)\frac{\partial}{\partial x^i} + c(t,x), \quad i,j = 1,\dots,d,$$

with real coefficients (written with the usual summation convention), f and g are given real-valued functions, and $T \in (0,\infty)$ is a constant. We assume that operator $\partial/\partial t - L$ is uniformly parabolic, and allow the growth in the spatial variables of the first and second-order coefficients in L (linear and quadratic growth, respectively), and of the data f and g (polynomial growth).

Multidimensional PDE problems arise in Financial Mathematics and in Mathematical Physics. We are mainly motivated by the application to a large class of stochastic models in Financial Mathematics comprising the non path-dependent options, with fixed exercise, written on multiple assets (basket options, exchange options, compound options, European options on future contracts and foreign-exchange, and others), and also to a particular type of path-dependent options: the Asian options (see, e.g., [14]).

Let us consider the stochastic modeling of a multi-asset financial option of European type under the framework of a general version of Black-Scholes model, where the vector of asset appreciation rates and the volatility matrix are taken time and space-dependent. Owing to a Feynman-Kač type formula, pricing this option can be reduced to solving the Cauchy problem (with terminal condition) for a degenerate second-order linear parabolic PDE of nondivergent type, with null term and unbounded coefficients (see, e.g., [14]). Therefore, alternatively to approximating the option price with probabilistic numerical methods, we can approximate the solution of the corresponding PDE problem with the use of non-probabilistic techniques.

When problem (1) is considered in connection with the Black-Scholes modeling of a financial option, we have that the growth of the vector SDE coefficients in the underlying financial model is appropriately matched. Also, by setting the problem with this generality, we cover the general case where the asset appreciation rate vector and the volatility matrix are taken time and space-dependent. Finally, by imposing weak conditions on the initial data g, we allow the financial derivative

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pay-off to be specified in a large class of functions. The free term f is included to further improve generality.

In the present study, we tackle the challenge posed to the spatial approximation[1] by the unboundedness of the PDE coefficients, under the strong assumption that the PDE does not degenerate. In order to facilitate the approach, we avoid numerical method sophistication, and make use of basic one-step finite-difference schemes.

The numerical methods and possible approximation results are strongly linked to the theory on the solvability of the PDEs. We make use of the L^2 theory of solvability of linear PDEs in weighted Sobolev spaces. In particular, we consider the PDE solvability in a class of weighted Sobolev spaces, the so-called well-weighted Sobolev spaces, first introduced by O. G. Purtukhia[2], and further generalised by I. Gyöngy and N. V. Krylov (see [10]), for the treatment of linear SPDEs. By constructing discrete versions of these spaces, we set a suitable discretised framework, and investigate the PDE approximation in space with the use of standard variational techniques.

We emphasize some points.

Firstly, we note that many PDE problems related to Finance are Cauchy problems: initial-boundary value problems arise mostly after a localization procedure for the purpose of obtaining implementable numerical schemes. Therefore, we do not find in many of these problems the complex domain geometries which are one important reason to favour other numerical methods (e.g., finite-element methods).

Also, although the finite-difference method for approximating PDEs is a well developed area, and the theory could be considered reasonably complete since three decades ago[3] [4], some important research is still currently pursued (see, e.g., the recent works [12, 13]).

Secondly, we observe that the usual procedure for obtaining implementable numerical schemes for problem (1) is to localize it to a bounded domain in $[0,T]\times\mathbb{R}^d$, and then to approximate the localized problem (see, e.g., [1, 16, 18]); see also [3, 17], where the approximation is pursued for more complex financial models but using the same localization technique). In this case, there is no need to consider weighted functional spaces for the solvability and approximation study, as the PDE coefficients are bounded in the truncated domain.

An alternative procedure is to (semi) discretise problem (1) in $[0,T]\times Z_h^d$, with Z_h^d the h-grid on $\mathbb{R}^d$, and then localize the discretised problem to a bounded domain in $[0,T]\times Z_h^d$, by imposing a discrete artificial boundary condition (see, e.g., [4,

[1] For the time discretisation, we refer to [8] where it is investigated the approximation of a general linear evolution equation problem which the PDE problem (1) can be cast into.

[2] The references for Purtukhia's works can be found in [10].

[3] We refer to [19] for a brief summary of the method's history, and also for the references of the seminal works by R. Courant, K. O. Friedrichs and H. Lewy, and further major contributions by many others.

[4] For the application of the finite-difference method to financial option pricing, we refer to the review paper [2] for the references of the original publications by M. Brennan and E. S. Schwartz, and further major research.

5, 6, 20], where several types of initial-value problems on unbounded domains are approximated; we refer to [5, 6, 20] for the procedure discussion). Our study is meaningful in this latter case, as the coefficient unboundedness remains a problem which must be dealt with.

Finally, we remark that: (i) the partial differential operators arising in Finance are of nondivergent type, and (ii) we do not assume the operator coefficients to be smooth enough to obtain an equivalent divergent operator. Therefore, although there are definitive advantages to considering the operator in the divergent form for the variational approach, this is not available for the present work.

With our study, we aim to contribute to the study of the numerical approximation of the general second-order parabolic problem (1), in the challenging case where the coefficients are unbounded (as well as the free data f and g). The results are obtained under weak regularity assumptions on the data. Also, an estimate for the rate of convergence of the discretised problem's generalised solution to the the exact problem's generalised solution is provided.

We outline the study. Firstly, we establish some well-known facts on the solvability of linear PDEs under a general framework, and introduce the well-weighted Sobolev spaces. Then, we discretise in space problem (1), with the use of finite-difference schemes. We set a discrete framework and, by showing that it is a particular case of the general framework previously presented, we deduce an existence and uniqueness result for the discretised problem's generalised solution. Finally, we investigate the approximation properties of the discrete scheme, and compute a rate of convergence.

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Using convexity methods for optimal stochastic switching

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Diverse problems in the area financial engineering can be frequently addressed as discrete-time stochastic control problems. For their solution, a variety of computational methods has been developed. However, the complexity of typical real-world applications usually goes beyond what is computationally feasible. In this talk, we address a novel method of approximate calculation of optimal control policy applicable to a particular class of control problems, whose stochastic dynamics exhibit a certain convexity preserving property. Utilizing this specific structure, we suggest a numerical algorithm which enjoys a number of desirable properties. Besides a very strong convergence properties, the main advantage of our approach is on the practical side, since we obtain an easy implementable scheme, based on simple matrix manipulations. We illustrate our method by applications in the portfolio optimization and to the investment decision optimization under partial information. Finally, we show how the estimate of the 'distance to optimality' of an approximative solution using Monte-Carlo based duality methods.

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On essential supremum and essential maximum with respect to random partial orders with applications to hedging of contingent claims under transaction costs

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In the theory of markets with proportional transaction costs a rather general model can be defined by an adapted (polyhedral) cone-valued process $\widehat{K} = (\widehat{K}_t)$ representing the solvency regions. The value processes are d-dimensional adapted processes and contingent claims are d-dimensional random variables or processes, see the book [1] where the "hat" notations are used to express assets in physical units as opposed to the "countability" in monetary terms, i.e. of units of the numéraire. Hedging a European-type contingent claim $\widehat{C} \in L(\mathbf{R}^d, \mathcal{F}_T)$ means to find a self-financing portfolio with the value process $\widehat{V} = (\widehat{V}_t)$ whose terminal value dominates the claim in the sense that the difference $\widehat{V}_T - \widehat{C}$ belongs to the random solvency cone $\widehat{K}_T$. Hedging an American-type contingent claim given by a pay-off process $\widehat{Y}$ is defined in an analogous way.

A practically important questions are: how to compute the hedging set of initial capitals admitting self-financing strategies super-replicating a given contingent claims and how to find such hedging strategies? A rather natural idea is to look for answers to these questions by studying the sets of "minimal" portfolios dominating the pay-offs. In the vector setting the concepts of minimality/maximality are not obvious and several analogs are suggested in the framework of vector and set-valued optimization usually in a deterministic framework. Placing the problem in a very general and abstract setting of a random partial order (or, more generally, a preference relation) we investigate seemingly new concepts of *Essssup* and *Essmax* and show that they are useful to define recursive relations for minimal hedging portfolios.

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Prior-to-default equivalent supermartingale measures

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1. Probabilistic set-up

Let E be a Polish space, modelling all possible states in an economy. Append a point $\triangle$ to E, which will model a "cemetery" state. If $\omega : [0, \infty) \mapsto E \cup \{\triangle\}$ is right-continuous, define $\zeta(\omega) := \inf\{t \in \mathbb{R}_+ \mid \omega(t) = \zeta\}$, with the interpretation of the economy's lifetime (or default). Define Ω as the set of all right-continuous $\omega : [0, \infty) \mapsto E \cup \{\triangle\}$ such that $\omega(0) \in E$ and $\omega(t) = \triangle$ holds for all $t \in [\zeta(\omega), \infty)$, and let $\mathbf{F} = (\mathcal{F}_t)_{t \in \mathbb{R}_+}$ be the right-continuous augmentation of the smallest filtration that makes the coordinate process on Ω adapted. Finally, set $\mathcal{F} := \bigvee_{t \in \mathbb{R}_+} \mathcal{F}_t$.

Two probabilities $\mathbb{P}$ and $\mathbb{Q}$ on $(\Omega, \mathcal{F})$ will be called *equivalent prior to* ζ if for any stopping time T and $A_T \in \mathcal{F}_T$, $\mathbb{Q}[A_T \cap \{T < \zeta\}] = 0$ holds if and only if $\mathbb{P}[A_T \cap \{T < \zeta\}] = 0$; denote such relationship via $\mathbb{Q} \sim_{<\zeta} \mathbb{P}$. Furthermore, agree to call ζ *foretellable under* $\mathbb{Q}$ if there exists a nondecreasing sequence $(\tau_n)_{n \in \mathbb{N}}$ of stopping times such that $\mathbb{Q}[\tau_n < \zeta, \forall n \in \mathbb{N}] = 1$ and $\mathbb{Q}[\lim_{n \to \infty} \tau_n = \zeta] = 1$. The next result, mathematically interesting in its own right, is of major importance (albeit, in a tacit way) in the development of the financial model later.

Theorem. For any probability measure $\mathbb{P}$ on $(\Omega, \mathcal{F})$, there exists $\mathbb{Q} \sim_{<\zeta} \mathbb{P}$ such that ζ is foretellable under $\mathbb{Q}$.

2. Financial set-up

The class of wealth processes available to an investor with (normalized) unit initial capital will be defined in a rather abstract and general-encompassing way. It contains as a special case any reasonable class of (potentially, constrained) nonnegative wealth processes resulting from frictionless trading that has appeared in the literature, including the possibility of an infinite number of assets (such as bond markets or equity markets with traded options), and allowing for overall default of the economy. A set $\mathcal{X}$ of stochastic processes will be called a *wealth-process set* if:

- Each $X \in \mathcal{X}$ is a nonnegative, adapted, right-continuous process with $X_0 = 1$, and $X_t = 0$ holds for all $t \geq \zeta$.
- $1_{[\zeta, \infty[} \in \mathcal{X}$.
- $\mathcal{X}$ is *fork-convex*: for any $s \in \mathbb{R}_+$, $X \in \mathcal{X}$, any strictly positive $X' \in \mathcal{X}$ and $X'' \in \mathcal{X}$, and any $[0, 1]$-valued $\mathcal{F}_s$-measurable random variable α_s, the process

$$\mathbb{R}_+ \ni t \mapsto \begin{cases} X_t, & \text{if } 0 \leq t < \zeta \wedge s, \\ \alpha_s (X_s / X'_s) X'_t + (1 - \alpha_s)(X_s / X''_s) X''_t, & \text{if } \zeta \wedge s \leq t < \zeta, \\ 0, & \text{if } \zeta \leq t, \end{cases} \quad \text{(FC)}$$

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is also an element of $\mathcal{X}$.

Fork-convexity corresponds to re-balancing: (FC) exactly describes the wealth generated when a financial agent invests according to X up to time s, and then reinvests a fraction α of the money in the wealth process described by X' and the remaining fraction $(1-\alpha)$ in the wealth process described by X''.

3. Fundamental Theorem of Asset Pricing (FTAP) and superhedging

For a stopping time T and nonnegative optional process V, define

$$p(V,T) := \inf\{x > 0 \mid \forall \epsilon > 0,\ \exists X^{x,\epsilon} \in x\mathcal{X} \text{ with } \mathbb{P}[X_T^{x,\epsilon} < V_T,\, T < \zeta] < \epsilon\}.$$

Loosely speaking, $p(V,T)$ is the minimal capital required at time zero in order to superhedge V_T at time T with as high a probability as desired, provided that default has not occurred by time T.

An *arbitrage of the first kind in the market* is a pair of $T \in \mathbb{R}_+$ and nonnegative optional process V such that $p(V,T) = 0$ and $\mathbb{P}[V_T > 0,\, T < \zeta] > 0$ hold. Say that condition NA_1 holds if there are *no* opportunities for arbitrage of the first kind.

Denote by $\mathcal{Q}$ the collection of all $\mathbb{Q} \sim_{<\zeta} \mathbb{P}$ such that all processes $X \in \mathcal{X}$ are (nonnegative) supermartingales.

FTAP. In the previous set-up, condition NA_1 is equivalent to $\mathcal{Q} \neq \emptyset$. Under any of the previous two (equivalent) conditions, for any stopping time T and nonnegative optional process V, the following *superhedging duality* holds:

$$p(V,T) = \sup_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}_{\mathbb{Q}}[V_T\,;\, T < \zeta].$$

4. Market completeness

For this last part, a precise mathematical statement will not be given, since it would require quite involved definitions and notation; however, we shall discuss below the main ideas in a somewhat precise manner.

In the setting of Section 2, assume further that $\mathcal{X}$ is closed in the semimartingale topology, locally in time for probabilities $\mathbb{Q} \sim_{<\zeta} \mathbb{P}$ under which ζ is foretellable. Then, under condition NA_1, one can obtain a variant of the second Fundamental Theorem of Asset Pricing, which provides an equivalence between the notion of market completeness (basically, the ability to perfectly replicate bounded nonnegative positions using *maximal* wealth processes) with uniqueness of a *maximal* probability $\mathbb{Q} \in \mathcal{Q}$. The latter qualification of maximality for $\mathbb{Q} \in \mathcal{Q}$ means that whenever $\overline{\mathbb{Q}} \in \mathcal{Q}$ is such that $\mathbb{Q}[A_T \cap \{T < \zeta\}] \leq \overline{\mathbb{Q}}[A_T \cap \{T < \zeta\}]$ holds for any stopping time T and $A_T \in \mathcal{F}_T$, then $\overline{\mathbb{Q}} = \mathbb{Q}$.

Investment and capital structure decisions under time-inconsistent preferences

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Based on a continuous-time model of quasi-hyperbolic discounting, this paper provides an analytically tractable framework of entrepreneurial firms' investment and capital structure decisions with time-inconsistent preferences. We show that the impact of time-inconsistent preferences on investment depends not only on the financing structures (all-equity financing or debt financing), but also on the entrepreneurs' belief regarding their future time-inconsistent behavior (sophisticated or naive). Time-inconsistent preferences delay investment under both all-equity financing and debt financing. However, the impact is weakened with debt financing, because debt financing increases the payoff value upon investment and accelerates investment. Naive entrepreneurs invest later and default earlier than sophisticated entrepreneurs, leading to a shorter operating period. Moreover, we find that naive entrepreneurs may choose higher leverage, while sophisticated entrepreneurs always choose lower leverage, compared to the time-consistent benchmark. These results support the empirical findings in entrepreneurial finance.

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Existence of endogenously complete equilibrium driven by diffusion

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The existence of complete Radner equilibria is established in an economy which parameters are driven by a diffusion process. Our results complement those in the literature. In particular, we work under essentially minimal regularity conditions and treat time-inhomogeneous case.

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Dynamic analysis of hedge fund returns: detecting leverage and fraud

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The subject of this talk is the application of dynamic analysis of financial time series data in detecting of fraud and other suspicious and de-stabilizing activity of financial institutions. Mr. Markov will present a powerful technique Dynamic Style Analysis (DSA) that helps to detect hidden risks, leverage and even alert to possible fraud using only financial performance data.

Hedge fund industry has grown rapidly over the past decade to over $2 trillion in assets and over 8,000 funds. At the same time, despite significant efforts to regulate the industry, the amount of information available to hedge fund investors remains negligible as compared to traditional investment products such as mutual funds. In most cases, the only available information on a hedge fund is a time series of monthly performance numbers and a vague description of the strategy. Hedge funds represent significant systemic risk: they amassed significant amounts of assets and through use of leverage and derivatives could destabilize world markets. In addition, some hedge funds manipulate their performance data and unsuspected investors become victims of outright fraud.

Practical aspects of the methodology will be discussed, e.g., parameter calibration, model selection, structural shift detection, etc.

The focus of the talk will be on case studies including:

- cases of fraud: a recent one (2012) in Japan where AIJ Investment Advisors defrauded pension funds of billions of dollars in a sophisticated derivatives strategy;

- detecting insider trading (Galleon hedge fund, 2009);

- massive arbitrage and rapid trading: Soros famous breaking of Bank of England (1992) and 2010 Flash Crash;

- extreme leverage: Long Term Capital (LCTM, 1998) where Federal Reserve had to orchestrate a bailout of the hedge fund to avoid disruption of world markets.

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Optimal stopping: a new approach with examples

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When considering optimal stopping problems we typically find two classes of results. The first one consists in the explicit solution to a concrete optimal stopping problem

$$V_\alpha(x) = \sup_\tau = \mathbf{E}_x e^{-\alpha\tau} g(X_\tau) = \mathbf{E}_x e^{-\alpha\tau^*} g(X_{\tau^*}). \tag{1}$$

Usually in this case one has to –somehow– guess the solution and prove that this guess in fact solves the optimization problem; we call this approach “verification”. For example we can consider the papers [7], [8], [11], [10]. The second class consists of general results, for wide classes of processes and reward functions. We call this the “theoretical” approach. It typically include results about properties of the solution. In this class we mention for example [2], [4], [1]. But these two classes not always meet, as frequently in concrete problems the assumptions of the theoretical studies are not fulfilled, and, what is more important, many of these theoretical studies do not provide concrete ways to find solutions. In what concerns the first approach, a usual procedure is to apply the *principle of smooth fit*, that generally leads to the solution of two equations: the *continuous fit* equation and the *smooth fit* equation. Once these equations are solved, a verification procedure is needed in order to prove that the candidate is the effective solution of the problem (see chapter IV in [9]). This approach, when an explicit solution can be found, is very effective. In what concerns the second approach, maybe the most important result is Dynkin's characterization of the solution of the value function V_α as the least α-excessive (or α-superharmonic) majorant of the payoff function g [2]. Other ways of classifying approaches in order to study optimal stopping problems include the martingale-Markovian dichotomy as exposed in [9].

Our departing point, inscribed in the Markovian approach, is Dynkin's characterization of the optimal stopping problem solution. Dynkin's characterization [2] states that, if the reward function is lower semi-continuous, V is the value function of the non-discounted optimal stopping problem with reward g if and only if V is the least excessive function such that $V(x) \geq g(x)$ for all $x \in \mathcal{I}$. Applying this result for the killed process Y, and taking into account the relation between X and Y, we obtain that V_α, the value function of the problem with discount α, is characterized as the least α-excessive majorant of g.

The second step uses Riesz's decomposition of an α-excessive function. We recall this decomposition in our context (see [5, 6, 3]). A function $u\colon \mathcal{I} \to \mathbb{R}$ is α-excessive if and only if there exist a non-negative Radon measure μ and an α-harmonic function such that

$$u(x) = \int_{(\ell,r)} G_\alpha(x,y)\mu(dy) + (\alpha\text{-harmonic function}). \tag{2}$$

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Furthermore, the previous representation is unique. The measure μ is called the representing measure of u.

The third step is based on the fact that the resolvent and the infinitesimal generator of a Markov process are inverse operators. Suppose that we can write

$$V_\alpha(x) = \int_{\mathcal{I}} G_\alpha(x,y)(\alpha - L)V_\alpha(y)m(dy), \tag{3}$$

where L is the infinitesimal generator and $m(dy)$ is the speed measure of the diffusion. Assuming that the stopping region has the form $\mathcal{I} \cap \{x \geq x^*\}$, and taking into account that V_α is α-harmonic in the continuation region and $V_\alpha = g$ in the stopping region we obtain as a suitable candidate to be the representing measure

$$\mu(dy) = \begin{cases} 0, & \text{if } y < x^*, \\ k\delta_{x^*}(dy), & \text{if } y = x^*, \\ (\alpha - L)g(y)m(dy), & \text{if } y > x^*, \end{cases} \tag{4}$$

Based on these considerations, we present some theoretical results and some new examples.

An important byproduct of our approach has to do with the smooth fit principle. Our results are independent of this principle, but they give sufficient conditions in order to guarantee it. Our approach is also applicable to certain classes of processes with jumps.

All the presented results are joint work with Fabián Crocce.

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Lower and upper bounds for Asian-type options: a unified approach

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In the context of dealing with financial risk management problems it is desirable to have accurate bounds for option prices in situations when pricing formulae do not exist in the closed form. A unified approach for obtaining upper and lower bounds for Asian-type options is proposed in this talk. The bounds obtained are applicable to the continuous and discrete-time frameworks for the case of time-dependent interest rates. Numerical examples will be provided to illustrate the accuracy of the bounds.

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A new stochastic Fubini theorem for measure-valued processes

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We prove a new stochastic Fubini theorem in a setting where we integrate measure-valued stochastic processes with respect to a d-dimensional martingale. To that end, we develop a notion of measure-valued stochastic integrals. As an application, we show how one can handle a class of quite general stochastic Volterra semimartingales.

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Robust hedging, price intervals and optimal transport

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The original transport problem is to optimally move a pile of soil to an excavation. Mathematically, given two measures of equal mass, we look for an optimal map that takes one measure to the other one and also minimizes a given cost functional. Kantorovich relaxed this problem by considering a measure whose marginals agree with given two measures instead of a bijection. This generalization linearizes the problem. Hence, allows for an easy existence result and enables one to identify its convex dual.

In robust hedging problems, we are also given two measures. Namely, the initial and the final distributions of a stock process. We then construct an optimal connection. In general, however, the cost functional depends on the whole path of this connection and not simply on the final value. Hence, one needs to consider processes instead of simply the maps S. The probability distribution of this process has prescribed marginals at final and initial times. Thus, it is in direct analogy with the Kantorovich measure. But, financial considerations restrict the process to be a martingale Interestingly, the dual also has a financial interpretation as a robust hedging (super-replication) problem.

In this talk, we prove an analogue of Kantorovich duality: the minimal super-replication cost in the robust setting is given as the supremum of the expectations of the contingent claim over all martingale measures with a given marginal at the maturity. The related papers are [1, 2].

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The pricing model of corporate securities under cross-holdings of equities and debts

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1. Model Setup

Our model is an extension of [1], [3], [2] and other related papers by introducing default costs.

Suppose that there are totally n firms that cross-hold their equities and debts and that the debts have a seniority structure with at most m priorities. Let b_i^k be the face value of k-th subordinated debt issued by firm i, $k = 1, \ldots, m$ and $\mathbf{b}^k = (b_1^k, \ldots, b_m^k)^\top$. That is, b_i^m is paid with first priority at maturity, b_i^{m-1} with second, etc.

The cross-holding structure of k-th debts is described by the $n \times n$ matrix $\mathbf{M}^k = (M_{ij}^k)_{i,j=1}^n$. More concretely, firm i owns a proportion $M_{ij}^k \in [0, 1]$ of the k-th debt issued by firm j ($M_{ii} = 0$). If $\mathbf{M}^k$ is a substochastic matrix, it means that a part of k-th debts is owned by outside investors. The equities are also cross-held with the structure $\mathbf{M}^0 = (M_{ij}^0)_{i,j=1}^n$.

Let e_i be the business asset of firm i and $\mathbf{e} = (e_1 \ldots, e_n)^\top$. The total asset (payment resource for debts) of firm i, denoted by a_i, is written as

$$a_i = e_i + \sum_{\ell=0}^{m} \sum_{j=1}^{n} M_{ij}^\ell r_j^\ell, \tag{1}$$

where r_i^k is the payoff of firm i's k-th debt at maturity for $k = 1, \ldots, m$ and r_i^0 is the payoff of firm i's equity ($\mathbf{r}^k = (r_1^k, \ldots, r_n^k)^\top$).

A key assumption in this study is that a default at maturity accompanies some liquidation costs. More concretely, if firm i cannot fully pay back all the debts at maturity, the payment resource of firm i is reduced by the proportion $\delta_i \in [0, 1]$. We write $\boldsymbol{\Delta} = \mathrm{diag}(\delta_i)_{i=1}^n$.

Define

$$d_i^k = \sum_{\ell=k+1}^{m} b_i^\ell \tag{2}$$

and $\mathbf{d}^k = (d_1^k, \ldots, d_n^k)^\top$. The clearing payment vector $\mathbf{r} \in \boldsymbol{R}^{(m+1)n}$ is naturally defined by the following equations:

$$r_i^0 = \left(a_i - d_i^0\right)_+ \tag{3}$$

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and

$$r_i^k = 1_{\{a_i \geq d_i^0\}} b_1^k + 1_{\{a_i(\mathbf{r}) < d_i^0\}} \left(b_i^k \wedge \left((1-\delta_i) a_i - d_i^k \right) \right)_+ \tag{4}$$

for $k = 1, \ldots, m$ and $i = 1, \ldots, n$.

2. Existence of Clearing Vectors

To study the clearing payment vector, let us define the function $\mathbf{f} : \boldsymbol{R}^{(m+1)n} \to \boldsymbol{R}^{(m+1)n}$ by

$$\mathbf{f}^0(\mathbf{r}) = \left(\mathbf{a}(\mathbf{r}) - \mathbf{d}^0\right)_+, \tag{5}$$

$$\mathbf{f}^k(\mathbf{r}) = (\mathbf{I} - \mathbf{D}(\mathbf{r}))\mathbf{b}^k + \mathbf{D}(\mathbf{r}) \left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{r}) - \mathbf{d}^k \right) \right)_+ \tag{6}$$

for $k = 1, \ldots, m$, where

$$\mathbf{a}(\mathbf{r}) = \mathbf{e} + \sum_{\ell=0}^{m} \mathbf{M}^\ell \mathbf{r}^\ell, \tag{7}$$

$$\mathbf{D}(\mathbf{r}) = \begin{pmatrix} 1_{\{a_1(\mathbf{r}) < d_1^0\}} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1_{\{a_n(\mathbf{r}) < d_n^0\}} \end{pmatrix}. \tag{8}$$

The clearing payment vector is expressed as a fixed point $\mathbf{f}(\mathbf{r}) = \mathbf{r}$.

Proposition 1. *Assume that* $\mathbf{M}^0$ *is substochastic. Then, there exists a vector* $\mathbf{r} \in \boldsymbol{R}_+^{(m+1)n}$ *such that* $\mathbf{f}(\mathbf{r}) = \mathbf{r}$.

Proof. Let the vector $\overline{\mathbf{r}}_0^0 \in \boldsymbol{R}_+^n$ be given by

$$\overline{\mathbf{r}}_0^0 = (\mathbf{I} - \mathbf{M}^0)^{-1} \left(\mathbf{e} + \sum_{\ell=1}^{m} \mathbf{M}^\ell \mathbf{b}^\ell - \mathbf{d}^0 \right)_+ \tag{9}$$

and define the sequence of the $(m+1)n$-dimensional vector $\{\overline{\mathbf{r}}_h\}$by

$$\overline{\mathbf{r}}_0 = (\overline{\mathbf{r}}_0^0, \mathbf{b}^1, \ldots, \mathbf{b}^m)^\top \tag{10}$$

and $\overline{\mathbf{r}}_h = \mathbf{f}(\overline{\mathbf{r}}_{h-1})$ for $h \geq 1$. We can verify that $\{\overline{\mathbf{r}}_h\}$ is a monotonically non-increasing sequence and bounded below. The limit $\overline{\mathbf{r}}$ is what we want to obtain. □

Alternatively, we can prove the existence of a clearing vector with the sequence $\{\underline{\mathbf{r}}_h\}$ defined by $\underline{\mathbf{r}}_0 = \mathbf{0}$ and $\underline{\mathbf{r}}_h = \mathbf{f}(\underline{\mathbf{r}}_{h-1})$. The proof is completed by noticing that $\{\underline{\mathbf{r}}_h\}$ is a non-decreasing sequence and bounded above, indicating the existence of a limit $\underline{\mathbf{r}}$. We can also show that $\overline{\mathbf{r}} \neq \underline{\mathbf{r}}$ in general.

Proposition 2. *The clearing payment vector* $\overline{\mathbf{r}}$ *is the greatest clearing vector in the sense that*

$$\mathbf{r} \leq \overline{\mathbf{r}} \tag{11}$$

for any $\mathbf{r} \in \boldsymbol{R}_+^{(m+1)n}$ *with* $\mathbf{f}(\mathbf{r}) = \mathbf{r}$*. Similarly the clearing payment vector* $\underline{\mathbf{r}}$ *is the least clearing vector in the sense that*

$$\mathbf{r} \geq \underline{\mathbf{r}} \tag{12}$$

for any clearing vector $\mathbf{r} \in \boldsymbol{R}_+^{(m+1)n}$.

Intuitively, the computational iteration $\{\overline{\mathbf{r}}_h\}$ echoes the *fictitious default algorithm* proposed by [1]. To see that, let the sequence of matrices $\{\overline{\mathbf{D}}_h\}$ be given by

$$\overline{\mathbf{D}}_0 = \mathbf{O}, \quad \overline{\mathbf{D}}_h = \mathbf{D}(\overline{\mathbf{r}}_h). \tag{13}$$

We use $\overline{\mathbf{D}}_0$ in the calculation of the vector $\overline{\mathbf{r}}_1$. In other words, we calculate the initial payment vector as if no firm defaults. After having $\overline{\mathbf{r}}_1$, we should calculate $\overline{\mathbf{D}}_1 = \overline{\mathbf{D}}(\overline{\mathbf{r}}_1)$ and check whether $\overline{\mathbf{D}}_1 = \overline{\mathbf{D}}_0$ or not. If the equality holds, $\overline{\mathbf{r}}_1$ is actually the greatest clearing vector and the default matrix is given by $\overline{\mathbf{D}}_0 = \overline{\mathbf{D}}_1 = \mathbf{O}$. Otherwise, we need to recalculate the payment vector $\overline{\mathbf{r}}_2$ with default matrix $\overline{\mathbf{D}}_1$. The iteration ends when $\overline{\mathbf{D}}_h = \overline{\mathbf{D}}_{h-1}$, determining which firms actually default. The greatest clearing vector is equal to $\overline{\mathbf{r}}_h$. A similar discussion can be applied to the derivation of $\underline{\mathbf{r}}$. That is, we set

$$\underline{\mathbf{D}}_0 = \mathbf{I}, \quad \underline{\mathbf{D}}_h = \mathbf{D}(\underline{\mathbf{r}}_h). \tag{14}$$

and calculate the sequence $\{\underline{\mathbf{r}}_h\}$. The survival firms are determined by the matrix $\mathbf{D}(\underline{\mathbf{r}})$. We call the process of $\{\underline{\mathbf{D}}_h\}$ the *fictitious survival algorithm.*

3. Numerical Results

If any.

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Semimartingale models with additional information and their applications in mathematical finance

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Semi-martingale models are widely used for pricing and hedging of financial assets. Namely, the price of risky asset can be represented by Dolan-Dade exponential $S = (S_t)_{t\geq 0}$ of some semi-martingale X:

$$S_t = S_0\,\mathcal{E}(X)_t.$$

In many cases the answer on the questions "what is the price" of the option related with S, and "what is the optimal strategy", depend on the information which an investor can have. Initiated by Baudoin (2003), and investigated in a number of papers (see, for example, Gasbarra, Valkeila, Vostrikova (2004), Hillairet, Jiao (2010,2011)) these questions begin to be very important in Mathematical Finance. According to the quantity of information of investors, we will distinguish three type of them, namely, non-informed, partially informed and perfectly informed agents. Since a semi-martingale is always given on filtered probability space, we will model the information of an investor by enlargement of the filtration. Namely, we suppose that this information is given by an additional random variable or a random process $\xi = (\xi_t)_{t\geq 0}$. Then, non-informed agents will work on initial probability space equipped with natural filtration $\mathbf{F} = (\mathcal{F}_t)_{t\geq 0}$ of the process X where

$$\mathcal{F}_t = \bigcap_{s>t} \sigma(X_u, u \leq s).$$

The partially informed agents will work on enlarged probability space with progressively enlarged filtration $\tilde{\mathbf{F}} = (\tilde{\mathcal{F}}_t)_{t\geq 0}$ where

$$\tilde{\mathcal{F}}_t = \bigcap_{s>t} \sigma(X_u, u \leq s) \otimes \sigma(\xi_u, u \leq s).$$

And, finally, the perfectly informed agents will use again enlarged probability space with initially enlarged filtration $\mathbf{G} = (\mathcal{G}_t)_{t\geq 0}$ where

$$\mathcal{G}_t = \bigcap_{s>t} \sigma(X_u, u \leq s) \otimes \sigma(\xi_u, u \leq T)$$

and T is time horizon. It should be pointed out that the enlargement of the filtration leads to incomplete markets even in the case when initial market was complete.

We give now a several examples of the situation presented above. The first one concerns default models. The modelling of defaultable world suppose to introduce

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a default time ξ, which can be simply a random time, related with some economical reasons such as structural changes, increasing of the price of raw material, or political changes, as change of the regime (see for instance, Eliott, Jeanblanc, Yor (2001), El Karoui, Jeanblanc, Jiao (2009)).

The next example concerns so called change-point models, i.e. the models with different behaviour before and after some random time. It should be noticed that change-point problems have a long history, probably beginning with the papers of Page (1955) in an a-posteriori setting, and of Shiryaev (1963) in a quickest detection setting. The problem was later considered in many papers and books, and was often related to a quickest detection approach. We underline, that not only quickest detection approach is interesting in financial mathematics, and this fact is related, for instance, with the investigation of the models with changing of the regime (see Cawston, Vostrikova (2011)) , in particular with the models with random dividends (see Gapeev, jeanblanc (2010)).

We will concentrate ourselves on the third example, which concerns the indifference pricing. Namely, in the real financial market investors can held traded risky assets of maturity time T and receive some particular derivatives such as contingent claims offering some pay-off at maturity time $T' > T > 0$. It can happen that the assets related with contingent claims can not be traded since the trading is difficult or impossible for investor because of lack of liquidity or legal restrictions. In this situation the investor would like maximize expected utility of total wealth and at the same time reduce the risk due to the uncertainty of pay-off of the contingent claim. In such situations the utility indifference pricing become to be a main tool for option pricing.

To be more precise, let us suppose that our market consists on non-risky asset $B_t = B_0 \exp(rt)$, where r is interest rate, and two risky assets

$$S_t = S_0\, \mathcal{E}(X)_t,\ \ \tilde{S}_t = \tilde{S}_0\, \mathcal{E}(\tilde{X})_t$$

where X and $\tilde{X}$ are semi-martingales with jumps $\Delta X > -1$, $\Delta \tilde{X} > \ -1$, and $\mathcal{E}$ is Dolean-Dade exponential. The investor can trade S and at the same time he has a European type claim on $\tilde{S}$ given by $g(\tilde{S}_{T'})$ where g is some real-valued Borel function. Let us denote by Π the set of self-financing predictable strategies. Then, for utility function U and initial capital x, the optimal expected utility related with S will be

$$V_T(x) = \sup_{\phi \in \Pi} E\left[U\left(x + \int_0^T \phi_s\, dS_s\right)\right]$$

and if we add an option, then the optimal utility will be equal to

$$V_T(x, g) = \sup_{\phi \in \Pi} E\left[U\left(x + \int_0^T \phi_s\, dS_s\ +\ g(\tilde{S}_{T'})\right)\right]$$

As known, the indifference price p_T^b for buyer of the option $g(\tilde{S}_{T'})$ is a solution to the equation

$$V_T(x - p_T^b, g) = V_T(x)$$

and it is an amount of money which the investor would be willing to pay today for the right to receive the claim and such that he is no worse off in expected utility terms then he would have been without the claim. The indifference price for the seller p_T^s of the option is a solution to the equation

$$V_T(x + p_T^b, -g) = V_T(x)$$

and it is an amount of money which the seller of the option would be willing to receive in counterpart of the option in order to preserve his own optimal utility.
The optimal utility of assets containing the options highly depends on the level of information of the investor about $\tilde{S}$. More precisely, the investor can be non-informed, partially informed or perfectly informed agent and the level of information changes the class Π mentioned in previous formulas. Namely, a non-informed agent can maximize his expected utility taking the strategies only from the set of self-financing predictable strategies with respect to the natural filtration $\mathbf{F}$ of X. At the same time, a partially informed agent can built his optimal strategy using the set of self-financing predictable strategies with respect to the progressively enlarged filtration $\tilde{\mathbf{F}}$ with the process $\tilde{X}$. Finally, a perfectly informed agent can use the self-financing predictable strategies with respect to initially enlarged filtration $\mathbf{G}$ with $\tilde{S}_{T'}$.

The first part of our talk is devoted to the general results about the maximisation of utility for semi-martingale models depending on a random factor. As previously let us introduce the total utility with the option $g(\xi)$:

$$V(x, g) = \sup_{\varphi \in \Pi(\mathbf{G})} E_{\mathbb{P}} \left[U \left(x + \int_0^T \varphi_s dS_s(\xi) + g(\xi) \right) \right]$$

Here $\Pi(\mathbf{G})$ is the set of all self-financing and admissible trading strategies related with the initially enlarged filtration $\mathbf{G} = (\mathcal{G}_t)_{t \in [0,T]}$, where $\mathcal{G}_t = \bigcap_{s>t} (\mathcal{F}_s \otimes \sigma(\xi))$. To solve the utility maximisation problem in the initially enlarged filtration we make an assumption about the absolute continuity of the conditional laws $\alpha^t = \mathbb{P}(\xi \mid \mathcal{F}_t)$ of the random variable ξ given $\mathcal{F}_t$ with respect to α, namely

$$\alpha^t \ll \alpha$$

for $t \in]0, T]$. Then we define the conditional laws $(P^u)_{u \in \Xi}$ of our semi-martingale $S(\xi)$ given $\{\xi = u\}$ and we reduce the initial utility maximisation problem to the conditional utility maximisation problem on the asset prices filtration $\mathbf{F}$. More precisely, to solve the utility maximisation problem on the enlarged filtration it is enough to solve the conditional utility maximisation problem on the asset prices filtration $\mathbf{F}$

$$V^u(x, g) = \sup_{\varphi \in \Pi^u(\mathbf{F})} E_{P^u} \left[U \left(x + \int_0^T \varphi_s(u) dS_s(u) + g(u) \right) \right]$$

and then integrate the solution with respect to α. It should be noticed that the investigation of semi-martingales depending of a random parameter leads, in general, to deep measurability problems as it was pointed out in Stricker, Yor (1978).

Finally, to solve conditional utility maximisation problem we use dual approach. Let us denote by f a convex conjugate of U. Under the assumption about the existence of an equivalent f-divergence minimal measure for the conditional semi-martingale model, we give the expression for conditional maximal utility. Then we give the final result for general utility maximisation problem.

As a next step, we do our study for HARA utilities. We introduce corresponding information quantities as entropies and Hellinger distances, and we give the expression for the maximal expected utility in terms of these quantities. Finally, we introduce the information processes, like Kullback-Leibler and Hellinger processes, and we give the expression of the maximal expected utility involving these information processes.

Using previous results, we give the explicite formulas for indifference price of buyers and sellers of the option for HARA utilities. Then we discuss risk measure properties of the mentioned indifference prices. We show that $-p_T^b(g)$ and $p_T^s(g)$ are risk measures.

As a particular case, we study utility maximisation and utility indifference pricing of exponential Levy models. It should be noticed that in Levy models case the information processes are deterministic processes containing the constants which are the solutions of relatively simple integral equations. It gives us the possibility to calculate the indifference prices relatively easy.

We apply identity in law technique to give the explicit calculus of information quantities for Geometric Brownian motion model. Then, previous results can be appied and it gives us the explicit formulas for indifference price in Geometric Brownian motion case.

Response to Paul A. Samuelson letters and papers on the Kelly capital growth investment strategy

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The Kelly Capital Growth Investment Strategy (KCGIS) is to maximize the expected utility of final wealth with a logarithmic utility function. This approach dates to Bernoullis 1738 suggestion of log as the utility function arguing that marginal utility was proportional to the reciprocal of current wealth. In 1956 Kelly showed that static expected log maximization yields the maximum asymptotic long run growth. Later, others added more good properties such as minimizing the time to large asymptotic goals, maximizing the median, and being ahead on average for the first period. But there are bad properties as well such as extremely large bets for short term favorable investment situations because the Arrow–Pratt risk aversion index is near zero. Paul Samuelson was a critic of this approach and here we discuss his various points sent in letters to Ziemba and papers reprinted in the recent book, MacLean, Thorp and Ziemba (2011). Samuelsons opposition has prevented many finance academics and professionals from using and suggesting Kelly strategies to students. For example, Ziemba was asked to explain this to Fidelity Investments, a major Boston investment firm close to and influenced by Samuelson at MIT. I agree that these points of Samuelson are theoretically correct and respond to theory. I argue that they all make sense and caution users of this approach to be careful and understand the true characteristics of these investments including ways to lower the investment exposure. While Samuelsons objections help us understand the theory better, they do not detract from numerous valuable applications, some of which are discussed here.

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Contributed talks

Option pricing via stochastic volatility models: impact of correlation structure on option prices

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In this talk, we discuss stochastic volatility models which play an important role in option pricing theoretically as well as practical implementation point of view. Here, we survey few well know stochastic volatility models and discuss three models governed by square root process (Heston 1993), Ornsten-Uhlenbeck process (Schobol-Zhu 1999) and double square root process (Zhu 2000) in detail. We estimate model parameters by utilizing India VIX data. We treat stochastic volatility as risk factor in option pricing dynamics and incorporate it in option pricing framework via characteristic functions of the Fourier transforms. We calculate option prices for different strike prices by considering that volatility is correlated with underlying stock prices and observe that there is significant correction in option prices after incorporation of stochastic volatility in the new framework. We analyse the behaviour of models on the monyness criteria of options.

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Pricing foreign currency options under jumps diffusions and stochastic interest rates

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Foreign exchange options are studied in the Heston stochastic volatility model for the exchange rate which includes jumps in both the spot exchange rate and volatility dynamics combined with the Cox , Ingersoll and Ross dynamics for the domestic and foreign stochastic interest rates. The instantaneous volatility is correlated with the dynamics of the exchange rate return, whereas the domestic and foreign short-term rates are assumed to be independent of the dynamics of the exchange rate. The main result furnishes a semi-analytical formula for the price of the foreign exchange European call option. The FX options pricing formula is derived using the probabilistic approach, which leads to explicit expressions for conditional characteristic functions. We argue that the model examined in this paper is the only analytically tractable version of the foreign exchange market model which includes jumps in the Heston stochastic volatility model and the exchange rate with the CIR dynamics for interest rates.

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Systemic risk with central counterparty clearing

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The reform of the functioning of over the counter (OTC) derivatives markets lies at the core of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010. Among the regulations is that the majority of OTC derivatives, of the order of hundreds of trillions of US dollars in terms of notional, should be centrally cleared so as to insure financial stability. The Basel Committee for Banking Supervision, European and UK regulators have enacted similar proposals.

Introducing a central clearing counterparty (CCP) modifies the intermediation structure of the market: any financial obligation between members of the CCP is now intermediated by the CCP, while part of the members' liquidity is transferred to the CCP in form of guarantee fund contributions. A CCP would therefore increase exposure concentration in the market and the critical issue is whether this is accompanied with proper capitalization of the CCP, proper guarantee fund requirements and proper management of the guarantee fund.

In this work we study financial networks in a stochastic framework. We measure systemic risk in terms of a coherent valuation principle. The framework allows us to examine the effects on systemic risk and price contagion of multilateral clearing via a central clearing counterparty. We build on the framework introduced by [3] and use a network representation of the OTC market to analyze contagion effects without and with central clearing, while accounting for liquidation costs. We prove existence and uniqueness of an interbank payment equilibrium in conjunction with the price impact on external assets. We find that a CCP not always reduces systemic risk and provide sufficient conditions for the latter to hold. We also propose an optimal capitalization of a CCP based on game theoretic arguments. A real world calibrated numerical study illustrates our findings.

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An equilibrium model for commodity forward prices

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We consider a market model that consists of financial speculators, the producers and the consumers of a commodity. Producers trade the forward contracts to hedge the commodity price uncertainty, while speculators invest in these forward to diversify their portfolios. It is argued that the commodity equilibrium prices are the ones that clear out the market of spot and forward contracts. Assuming that producers and speculators are utility maximizers, and that the consumers demand and the exogenously given financial market are driven by a Lévy process, we provide closed-form solutions for the equilibrium prices and analyze their dependence on the model parameters. A dynamic version of this equilibrium model is also established and discussed.

The producers produce π_0 units of the commodity at the initial time 0 and π_T units at the terminal time T; both π_0 and π_T are assumed deterministic. At the initial time $t = 0$, the producers choose how much of the production π_0 are going to sell at the spot and how much are going to store in order to sell at T. The spot price of the commodity is determined by the demand function of the consumers. Although the initial demand function for the commodity is known (a given function $\psi_0(\cdot)$), the fluctuation of the commodity spot price at time T is caused by the randomness of the demand function at the terminal time T, denoted by $\psi_T(\cdot)$. Producers can hedge the risk of this price fluctuation by shorting forward contracts with maturity at T written on the commodity. Also, they have the option to maintain some of their initial production in storage and sell it at the terminal time T. Therefore, the representative producer's hedging/storage problem is

$$\sup_{\alpha\in[0,\pi_0],h^p\in\mathbb{R}} \mathbb{E}[\mathbb{U}_p(w(\alpha,h^p))] \tag{1}$$

where $w(\alpha,h^p) := P_0(\pi_0-\alpha)(1+R)+P_T(\pi_T+\alpha(1-c))+h^p(F-P_T)$ and $\mathbb{U}_p$ is her utility, P_0 and F are the commodity spot and forward prices (both endogenously determined), P_T is the commodity spot at time T, R is the interest of the risk-free investment in period $(0,T)$ and c is the storage cost considered as percentage.

One the other hand, speculators trade continuously in an exogenously priced stock market and satisfy the producers' hedging demand by longing the corresponding forward contracts

$$\sup_{\theta\in\Theta,h^s\in\mathbb{R}} \mathbb{E}\left[\mathbb{U}_s\left(h^s(P_T-F)+\int_0^T \theta_s dS_s\right)\right] \tag{2}$$

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where $(S_t)_{t\in[0,T]}$ is the stock price process (exogenously priced). We further assume that the market is driven by a vector of stochastic factors modelled by an d-dimensional Lévy process in the following way: $S_t = S_0\mathcal{E}(\langle u_1, X_t\rangle)$ and the consumers' random demand at time T is given by $\psi_T(P) = \psi_0(P) + \langle u_2, X_T\rangle$, where $u_i \in \mathbb{R}^d$, $i = 1, 2$.

The central goal of this paper is to determine the spot and forward prices of the commodity that makes the forward and the spot market clear out. Given the optimization problems (1) and (2), the equilibrium price of the forward contract on this commodity is the one that makes the forward market equilibrate, that is the price $\hat{F}$ that solves the equation $h^p(\hat{F}) = h^s(\hat{F})$. The corresponding price $\hat{P}_0$ is the equilibrium spot price.

A List of Contributions

The main results of this work can be summarized in the following list:

- The existence and the uniqueness of the equilibrium spot and forward prices are proved when agents' utility functions are exponential.
- We extensively analyze two market model examples; one with continuous stochastic factors and one with jumps. In both cases, we get closed-form solutions of the equilibrium prices and discuss the findings. These formulas allow us for instance to identify which parameters have an upward impact on the commodity spot price.
- A dynamic version of the this equilibrium problem is also established and developed.

On the optimal debt ceiling

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Motivated by the current debt crisis in the world, we consider a government that wants to control optimally its debt ratio. The debt generates a cost for the country. The government can reduce the debt ratio, but there is a cost associated with this reduction. We obtain a solution for the government debt problem. In particular, we obtain an explicit formula for the optimal debt ceiling.

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Fourier transform methods for pathwise covariance estimation in the presence of jumps

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With a view to calibrating multivariate stochastic covariance models, we provide a nonparametric method to estimate the trajectory of the instantaneous covariance process from observations of a d-dimensional logarithmic price process. We work under the mild structural assumption of Itô-semimartingales allowing in particular for a general jump structure. By means of this procedure we can determine certain quantities of general multivariate models from time series observations, even when considering the model under some pricing measure.

Our approach combines instantaneous covariance estimation based on Fourier methods, as proposed by Malliavin and Mancino [6, 7], with jump robust estimators for integrated covariance estimation [1, 2, 3, 4, 5, 8, 9]. We study in particular asymptotic properties and provide a central limit theorem, showing that, in comparison to the classical estimator of the instantaneous covariance, the asymptotic estimator variance of the Fourier estimator is smaller by a factor 3/2. The procedure is robust enough to allow for an iteration and we can therefore show theoretically and empirically how to estimate the integrated realized covariance of the instantaneous stochastic covariance process. For a large class of multivariate stochastic covariance models, this then allows to estimate quantities which remain invariant under equivalent measure changes, such as volatility of volatility, from time series observations. We can therefore apply these techniques to robust calibration problems for multivariate modeling in finance, i.e. the selection of a pricing measure by using time series and derivatives' price information simultaneously. "Robust" here means that re-calibration is more stable over time, that the estimation procedures of, e.g., instantaneous covariance also work in the presence of jumps, and that the procedures are as robust as possible with respect to input deficiencies.

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Local volatility models: approximation and regularization

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We aim at understanding the typical shape of a local volatility surface, by focusing on its extremes (w.r.t. strike and maturity). The asymptotic behavior is governed by a saddle-point based formula, akin to Lees moment formula for implied volatility. Applications include local vol parametrization design and assessing volatility model risk. Secondly, it is well known that local vol models cannot deal with jumps in the underlying. We propose a simple regularization procedure as a remedy. Its validity is related to recent work of Yor et al. on Kellerers theorem from the theory of peacocks (processus croissants pour lordre convexe). The talk is based on joint work with S. De Marco, P. Friz, and M. Yor.

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A moment matching market implied calibration

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The calibration of a model on a given market situation is a critical and important part of any derivative pricing and risk management exercise. Traditionally, one solves the so-called inverse problem, which consists of finding the parameter set that is compatible with the observed market price of a set of liquidly traded derivatives. Typically, a perfect match is not plausible and one looks for an "optimal match". More precisely, one minimizes the distance between the model and the market prices of benchmark instruments by using a search algorithm. Most commonly, practitioners are minimizing the root mean square error between model and market vanilla prices or between model and market implied volatilities. Although the root mean square objective function is the current industry practice, there exist other alternatives just as suitable. The optimal parameter set typically strongly depends on the choice of the objective function, leading to significantly different prices for the more exotic and structured products. Besides the calibration risk issue, it is well known and documented that several additional problems can arise with the standard calibration methodology. Firstly, one faces the problem of selecting an appropriate starting value for the search algorithm used. Indeed, the objective function to be minimized is typically a non-convex function of the model parameter set and can thus have several local minima, making the solution of the standard calibration problem dependent on the initial parameter set, which is taken as starting value of the optimization algorithm and on the sophistication of the numerical search performed. Further, one has the typical related problem of finding a local minimum instead of the global minimum. Also, a calibration exercise can be quite time consuming, especially if the number of parameters to be calibrated is becoming large.

Hence, we provide a new calibration formalism which consists of matching the moments of the asset log-return process with those inferred from liquid market data. In particular, we derive a model independent formula for the moments of the asset log-return distribution function by expanding power returns as a weighted sum of vanilla option payoffs. The new calibration methodology rests on closed-form formulae only: it is shown that, for a model with N parameters, the moment matching calibration problem reduces to a system of N algebraic equations which give directly the optimal parameter set in terms of the market implied standardized moments of order 2 to order N and avoids thus the delicate choice of a particular objective function. For the numerical study, we first work out different popular exponential Lévy models and illustrate how the new methodology outperforms the current market standard ones in terms of both the computation time and the quality of the fit. We then consider exponential Lévy models with piecewise constant

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parameters between successive quoted option maturities and extend the moment matching market implied calibration procedure to take into account the term structure characteristic of these models. More particularly, we propose a bootstrapping moment matching calibration. This sequential calibration arises naturally due to the additive property of cumulants of independent random variables and consists in solving M independent moment matching systems of N equations, where M denotes the number of quoted maturities.

The new calibration formalism provides thus an appealing alternative to the standard calibration problem since, for the Lévy models under investigation, the method is not requiring any search algorithm and hence any starting value for the model parameters, it is almost instantaneously delivering the matching parameters and it avoids local minima problems. The new methodology can also be used as a preliminary calibration aimed at delivering appropriate market implied starting values or prior model for the standard inverse problem.

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On a connection between superhedging prices and the dual problem in utility maximization

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1. Kramkov and Schachermayer in their seminal paper [3] studied the problem of maximizing the expected utility of terminal wealth in the framework of a general incomplete semimartingale model of a financial market. One of the main results of [3], Theorem 2.1, says that the Fenchel conjugate to the value function of the utility maximization problem is the value function of a dual problem, and this dual problem is a minimization problem over the set of the terminal values of supermartingale densities (deflators). This result was proved in two steps. At the first step an abstract version of this result, Theorem 3.1, was proved. Its statement deals with an abstract market model and says that the minimum in the dual problem has to be taken over a certain set $\mathscr{D}$. In a semimartingale model, this set $\mathscr{D}$ is larger than the set of the terminal values of supermartingale densities. At the second step it was shown that the value functions of the corresponding minimization problems over these two sets coincide. An analysis of the proof of the second step shows that it is based on Theorem 5.5 in Delbaen and Schachermayer [1] that says that the superhedging price of any nonnegative contingent claim can be computed as the supremum of its expectations over the set of all equivalent σ-martingale measures.

The aim of this paper is to understand on an abstract level what is the mechanism responsible for this connection.

2. Let $(\Omega, \mathscr{F}, \mathsf{P})$ be a probability space. Denote by L^0 the space of all (equivalence classes of) real-valued random variables. L^0 is equipped with the convergence in probability, and bar means the closure with respect to this convergence. L^0_+ is the cone in L^0 consisting of nonnegative random variables.

We consider an abstract market model described as a quadruple $(\Omega, \mathscr{F}, \mathsf{P}, \mathscr{A})$. where $\mathscr{A}$ is a convex subset of L^0_+. It is assumed also that $\mathscr{A}$ contains a random variable ξ such that $\mathsf{P}(\xi \geq \varkappa) = 1$ for some $\varkappa > 0$. $\mathscr{A}$ is interpreted as the set of terminal wealths of an investor corresponding to all her strategies with initial wealth 1. If the initial wealth is $x > 0$, then the corresponding set of terminal wealth is $x\mathscr{A}$.

Put $\mathscr{A}_0 = \left(\mathscr{A} - L^0_+\right) \cap L^0_+$, $\mathscr{C} = \overline{\mathscr{A}_0}$, $\mathscr{D} = \{\eta \in L^0_+ \colon \mathsf{E}\eta\xi \leq 1 \text{ for all } \xi \in \mathscr{A}\}$.

Let $B \in L^0_+$. A possible definition of the superhedging price of B is

$$\pi(B) = \inf\{x > 0 \colon \text{there is a } \xi \in \mathscr{A} \text{ such that } B \leq x\xi\} = \inf\{x > 0 \colon B \in x\mathscr{A}_0\}$$

(here and below $\inf \varnothing = +\infty$). Since we do not assume any kind of closedness of

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$\mathscr{A}$ here, an alternative (and more natural) definition of the superhedging price is

$$\pi_*(B) = \inf\{x > 0\colon B \in x\mathscr{C}\} = \sup_{\eta\in\mathscr{D}} \mathsf{E}\eta B.$$

Here the second equality follows from the bipolar theorem by Brannath and Schachermayer. Obviously, $\pi_*(B) \le \pi(B)$ for all $B \in L^0_+$. It is easy to check that

$$\pi(B) = \pi_*(B) \text{ for all } B \in L^0_+ \qquad \Longleftrightarrow \qquad \mathscr{C} \subseteq \bigcap_{\lambda>1} (\lambda\mathscr{A}_0).$$

3. Let $U\colon \mathbb{R} \to [-\infty, +\infty)$ be a utility function. More precisely, here we assume that U is concave, $U(x) \equiv -\infty$ on $(-\infty, 0)$ and $U(x) \in \mathbb{R}$ on $(0,\infty)$, and U is strictly increasing on $(0,\infty)$. No other assumptions on U are imposed. As usual,

$$V(y) = \sup_{x>0} [U(x) - xy], \quad y \in \mathbb{R}.$$

For a probability measure $\mathsf{Q} \ll \mathsf{P}$ define the value function $u_{\mathsf{Q}}(x)$, $x > 0$, in the utility maximization problem relative to Q and also the value function $v_{\mathsf{Q}}(y)$, $y \ge 0$, in the dual minimization problem:

$$u_{\mathsf{Q}}(x) = \sup_{\xi\in x\mathscr{A}} \mathsf{E}_{\mathsf{Q}} U(\xi), \qquad v_{\mathsf{Q}}(y) = \inf_{\eta\in\mathscr{D}} \mathsf{E}_{\mathsf{Q}} V\Big(\frac{y\eta}{d\mathsf{Q}/d\mathsf{P}}\Big).$$

As is shown im Kramkov and Schachermayer [3, Theorem 3.1], see also Gushchin [2, Theorem 2.2] for a refined version, the following dual relations hold:

$$u_{\mathsf{Q}}(x) = \min_{y\ge0} [v_{\mathsf{Q}}(y) + xy], \quad x > 0, \qquad v_{\mathsf{Q}}(y) = \sup_{x>0} [u_{\mathsf{Q}}(x) - xy], \quad y \ge 0.$$

Theorem 1. *Let $\mathscr{W}$ be a nonempty convex subset of $\mathscr{D}$.*

(i) *Assume that for a given utility function U, for all $\mathsf{Q} \ll \mathsf{P}$ and $y \ge 0$,*

$$v_{\mathsf{Q}}(y) = \inf_{\eta\in\mathscr{W}} \mathsf{E}_{\mathsf{Q}} V\Big(\frac{y\eta}{d\mathsf{Q}/d\mathsf{P}}\Big).$$

Then

$$\pi_*(B) = \sup_{\eta\in\mathscr{W}} \mathsf{E}\eta B \quad \text{for every } B \in L^0_+. \tag{1}$$

(ii) *Let (1) be satisfied. Then*

$$v_{\mathsf{Q}}(y) = \inf_{\eta\in\overline{\mathscr{W}}} \mathsf{E}_{\mathsf{Q}} V\Big(\frac{y\eta}{d\mathsf{Q}/d\mathsf{P}}\Big)$$

for all $\mathsf{Q} \ll \mathsf{P}$ and $y \ge 0$, and for every utility function U described above.

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What can be inferred from a single cross-section of stock returns?

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We develop a new econometric technique – the Generalized Method of Moments(GMM) under a non-localized common shock – to consistently estimate parameters of a financial market model using only a single cross-section of stock returns. Unlike the existing methods, the technique does not require observing a long history of stock return data. Our technique is novel in that it accounts for a non-localized common shock (e.g., market risk) affecting cross-sectional observations, which induces strong cross-sectional data dependence and renders standard estimation methods inconsistent. Also, it differs from the popular two-pass regression method of Fama and MacBeth (1973), which is consistent as the time-series length grows infinitely large. In contrast, our proposed GMM estimators are shown to be consistent as the number of stocks in a cross-section grows infinitely large. We also prove that the estimators are asymptotically mixed normal, which differentiates them from the regular, asymptotically normal GMM case, Hansen (1982). Despite the asymptotic mixed normality, statistical inference on parameters can still be conducted using conventional Wald tests. In addition, the overidentifying restrictions (OIR) test is shown to retain its standard properties. Until now, cross-sectional GMM estimation under a non-localized common shock has received only tangential attention in Andrews (2003), where such a possibility is suggested but not fully explored. To empirically illustrate the technique, we estimate a financial model that is an adaptation of the classical ICAPM model of Merton (1973). The model comprises a well-diversified market portfolio index and a cross-section of stocks. The price of the index follows a geometric Brownian motion and is affected by a single source of market risk. Individual stock prices follow geometric Brownian motions and depend on this same source of market risk, but are additionally affected by stock-specific idiosyncratic risks. We discuss in detail the estimation of an idiosyncratic volatility premium. Using cross-sections of daily, weekly, monthly, quarterly, and annual U.S. stock returns from 2000–2011, we find that the premium is positive on daily return data, but tends to be negative on monthly, quarterly, and annual data.

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Symbolic CTQ-analysis – a new method for studying of financial indicators

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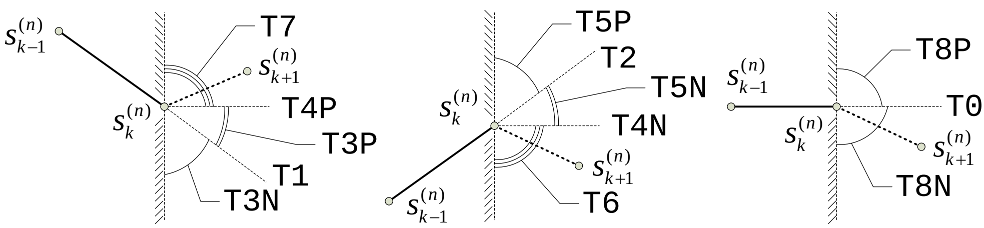
Modern financial and economic system extremely complex, and the creation of accurate adequate models from first principles is very difficult. One of the approaches to the study of these systems is based on the analysis of financial and economic time series of the form:

$$\{\mathbf{s}_k\}_{k=1}^{K}, \quad \mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N, \quad n = \overline{1,\, N}, \quad k \in \mathrm{K} \subset \mathbb{N}, \quad k = \overline{1,\, K}. \tag{1}$$

Every k-th countdown can be associated with moment of time t_k, at that $t_{k+1} > t_k$, $t \in \mathrm{T} \subset \mathbb{R}$. Variable $s_k^{(n)}$ – may be interpreted as value of some financial indicator on moment of time t_k.

In recent times, in financial mathematics for the analysis of these time series are increasingly using methods of statistical physics, nonlinear dynamics and chaos theory. One of the most effective tools - this a symbolic dynamics, which allows you to explore a variety of complex phenomena in dynamical systems: chaos, strange attractors, hyperbolic, structural stability, controllability, etc. In the author article [1] introduced by the finite T-alphabet for encoding shape of trajectories of $\{\mathbf{s}_k\}_{k=1}^{K}$ in the space $\mathrm{S} \times \mathrm{K}$ through the matching: $\{s_k^{(n)}\}_{k=0}^{K+1} \Rightarrow \{T_k^{\alpha\varphi}|_n\}_{k=1}^{K}$, $T_k^{\alpha\varphi} = \big[T_k^{\alpha\varphi}|_1 \ldots T_k^{\alpha\varphi}|_N\big]$. The scheme of terms $T^{\alpha\varphi}|_n$ is shown in the figure.



In addition, the approach allows us to analyze the level of synchronization and its temporal structure for complex ensembles of highly non-stationary and non-identical chaotic oscillators large dimensions with arbitrary shape and topology of the network (lattice) [2, 3].

This report illustrates the basic features of the symbolic CTQ-analysis applied to the study of the dynamics of financial indicators. As an example, we study the structure and parameters of the synchronization rates of world currencies (the U.S. Dollar [USD], Euro [EUR], Japanese Yen [JPH], Swiss Franc [CHF], and the British Pound [GBP]) against the ruble of the Russian Federation [RUB] for period from 01.01.1999 on 31.03.2013. The initial data are taken from the official web-site

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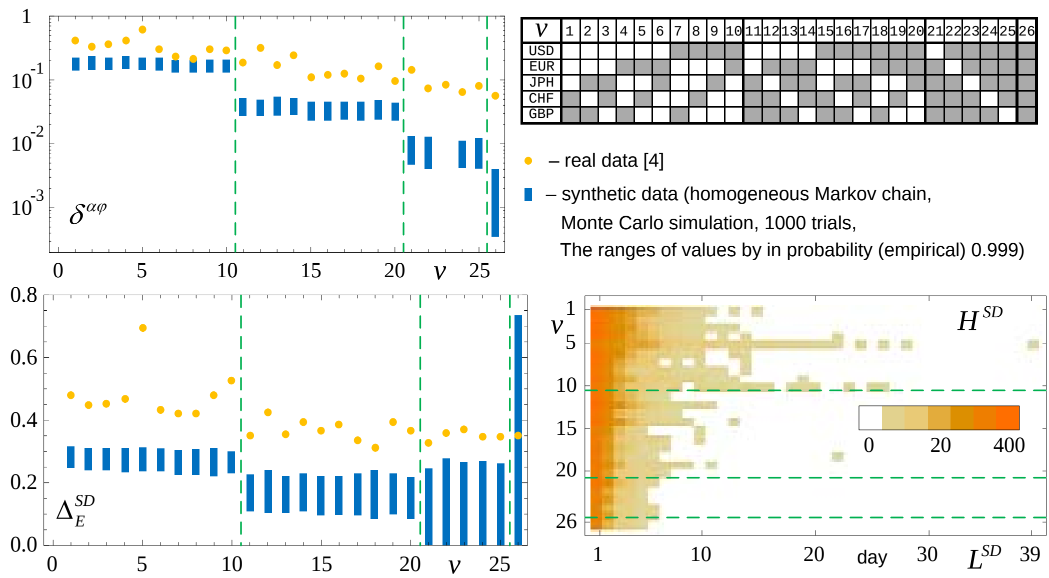
Central Bank of Russia [4]. The sample size of $K = 3\ 545$ counts. Data extraction and processing was carried out in the program Wolfram Mathematica 9.

Among the results obtained. For specified 5-financial indicators are constructed and analyzed their of symbolic TQ-images [1, 3]. The analysis revealed a difference between the structure of the time series for the exchange rate USD/RUB and the rest currency pairs. Over all possible combinations (v) the specified 5-currency pairs, the analysis of T-synchronization [2, 3]. For some combinations found non-random higher values of integral level of synchronicity $\delta^{\alpha\varphi}$ and the entropy of the structure of synchronous domains Δ_E^{SD}. In the H^{SD} spectra of synchronous domains, detected long periods L^{SD} of synchronized exchange rate fluctuations. For all relevant periods obtained their real date (for linking the to external events).



It is further planned multiscale CTQ-analysis of these financial indicators for investigation of their temporal structure in order to study the mechanism and causes of synchronicity.

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Cramér-von Mises test for Gauss processes

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One of the problems in the theory of the goodness-of-fit tests is the problem to test if an observed random process $S(t)$ on $[0,1]$ is the Gauss process with zero mean and a covariance function $K_S(t,\tau),\ t,\tau \in [0,1]$. This problem arises in particular in applications of financial mathematics, when the assumption that the process under study is a Gaussian process, is not deemed sufficient reasonable. This test should be based on n realisations $S_1(t), S_2(t), ..., S_n(t),\ t \in [0,1]$, of $S(t)$. The process $S(t)$ and its realisations are considered here as the elements of the separable Hilbert space $L^2([0,1])$. We choose as a basis for $L^2([0,1])$ the orthonormal basis formed by eigenfunctions $g_1(t), g_2(t), ...$ of the covariance operator with the kernel $K_S(t,\tau)$. The processes $S(t)$ and $S_i(t)$ can be represented in the form of expansion in the mentioned basis as $\mathbf{s} = (s_1, s_2, s_3, ...)$ and $\mathbf{s}_i = (s_{i1}, s_{i2}, s_{i3}, ...)$, correspondingly. The vector $\mathbf{s}$ has independent components with normal distributions. It can be transformed to the random vector $\mathbf{T} = (T_1, T_2, T_3, ...),\ \ \mathbf{T} \in [0,1]^\infty$, with the independent components, having the uniform distribution on $[0,1]$. The observations $\mathbf{s}_i$ can be transformed similarly to the observations $\mathbf{T}_i = (T_{i1}, T_{i2}, T_{i3}, ...)$ of the "uniform" distribution on $[0,1]^\infty$. It can be introduced a "distribution function"

$$F(\mathbf{t}) = F(t_1, t_2, t_3 ...) = P\{T_1 \le t_1^{\alpha_1}, T_1 \le t_1^{\alpha_1}, T_1 \le t_1^{\alpha_1} ...\} = t_1^{\alpha_1} t_2^{\alpha_2} t_3^{\alpha_3} ... \ .$$

Here, $\alpha_i > -1$ should tend sufficiently quickly toward zero. Correspondingly, the empirical distribution function can be introduced as

$$F_n(\mathbf{t}) = F_n(t_1, t_2, t_3 ...) = (1/n)\ \sharp\{\mathbf{T}_i : \ \ T_{i1} \le t_1^{\alpha_1},\ T_{i2}^{\alpha_2} \le t_2, ...\}.$$

The empirical process $\xi_n(\mathbf{t}) = \sqrt{(}n)(F_n(\mathbf{t}) - F(\mathbf{t})),\ \mathbf{t} \in [0,1]^\infty$, weakly converges to the Gaussian process in $L_2(L_2[0,1])$. This process has the zero mean and covariance function

$$K(\mathbf{t}, \mathbf{v}) = \prod_{i=1}^{\infty} \min(t_i^{\alpha_i}, v_i^{\alpha_i}) - \prod_{i=1}^{\infty} t_i^{\alpha_i} v_i^{\alpha_i}.$$

The Cramér-von Mises statistic

$$\omega_n^2 = n \int_{[0,1]^\infty} \left(\frac{1}{n} \sum_{i=1}^{n} \prod_{j=1}^{\infty} I_{T_{i,j} < t_j} - \prod_{i=1}^{\infty} t_i^{\alpha_i} \right) d\mathbf{t}.$$

Limit distribution of the Cramér-von Mises statistic is calculated using the methods described in the papers listed in the bibliography. The statistic ω_n^2 can be calculated by the Monte-Carlo method. In turn, the distribution of the statistic was calculated

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also using the Monte Carlo method. We will present the estimated quantiles of the distribution ω_n^2 with $\alpha_i = 1/i^3$. The integration was carried out over the cube $[0,1]^{10}$. The percent points are $P\{\omega_n^2 \leq 0.90\} = 0.34$ and $P\{\omega_n^2 \leq 0.95\} = 0.45$. It can be noted that corresponding percent points for the classical univariate Cramér-von Mises statistic are 0.35 and 0.46.

Described theory can be directly applied also to testing the distribution uniformity on the unit multidimensional cube.

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The value of Asian options in the Black-Scholes model: PDE approach

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The Asian option (or average value option) is a special type of option contract. The Asian options payoff is determined by the average underlying price over some pre-set period of time. There are several types of Asian options: fixed strike (also known as the average rate) Asian option payout based on the average price of the underlying asset, and the floating strike the so-called floating rate) Asian option payout (based on the average price and the spot price of the underlying asset. Note, that in almost all works about the Asian option value in the Black Scholes model the fixed strike options are considered. In this work we will consider both fixed and floating strike options. The results of this work summarize the results of papers [7], [8], [12] and [13]. This talk is about applying PDE methods for Asian options value determination. The main method in this work is the PDE approach: it is known that the value of an option is the expected value from the discount payout function, and this expected value is the solution of the parabolic PDE boundary problem. Using the Laplace transform, and the theory of hypergeometric and special functions we determine the Green function of the original problem. The report will show that the Laplace transform of the basic equation can be reduced to the Whittaker equation (The Schrdinger equation with Morse potential). Based on this fact, we construct the Green function as the infinite integral of the product of the first and second kind Whittaker functions (Confluent hypergeometric function). Using the properties of special functions we find two more representations of the Green function: the infinite integral of the first kind Macdonald function (Modified Bessel function) and the double infinite integral of elementary functions. Then, the option value is the convolution product of the Green function and the payout function. A big number of works originating from Boyle and Emmanuel [1] is devoted to the Asian option value problem in the Black-Scholes model. There are several directions dealing with this issue: the analytical methods, the estimates of the value, the Monte-Carlo simulations and the PDE numerical methods. The estimates of the interested value are covered by Turnbull and Wakeman [2], Milevsky and Posner[3] and other authors. The numerical approaches are considered Kemna and Vorst[4], Rogers and Shi [5] , Vecer [6] and other authors. The first work about analytical methods in the Asian option value problem was Yors [7] work published in 1992. In this work the value was found as the triple infinite integral. Later, in 1993 Geman and Yor[8] determined the Laplace transform of value, which can be inverted numerically. This technique is illustrated by Fu[10], Craddock[11] and other authors. In 2000 Dufresne determined the value as the infinite series of the Laguerre polynomials. Finally, Linetsky[13] using the spectral expansion methods

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found the value as the infinite integral of the Whittaker functions, which was estimated through a series of the same functions. We should mention that the Asian option value problem is partially covered by Yor and Matsumoto [14], [15].

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Sequential hypothesis testing for a drift of a fractional Brownian motion

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1. Let $X = (X_t)_{t\geq 0}$ be a random process defined on a probability space $(\Omega, \mathscr{F}, \mathsf{P})$ by the formula

$$X_t = \mu t + B_t^H, \tag{1}$$

where $B^H = (B_t^H)_{t\geq 0}$ is a fractional Brownian motion with Hurst index $H \in (0,1)$, and μ is a drift value. Recall that $B^H = (B_t^H)_{t\geq 0}$ is a Gaussian process starting from zero, having zero mean and covariance function

$$R(s,t) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}), \qquad s,t \geq 0.$$

It is well known that, for $H \in (0,1/2) \cup (1/2,1)$, such a process B^H is neither a semimartingale nor a Markov process (see, for example, [1]).

The problem of estimation of the parameters μ and H in the model (1) naturally arises in a context of modeling of stock prices [2]. In this note we suppose that the Hurst index H is known and μ is a random variable which is independent of B^H (we suppose that $\mathsf{E}\,|\mu| < \infty$). Thus, we follow the Bayesian approach.

2. The problem of a sequential *estimation* of the drift μ in such a setting was studied in [2]. Here we consider the problem of *testing the hypothesis* $H_1, \ldots, H_n$, $H_i\colon \mu \in A_i$, by a sequential observation of X. The sets $A_1, \ldots, A_n \subset \mathbb{R}$ are supposed to be nonintersecting and such that $\sum_i P(\mu \in A_i) = 1$.

Each testing procedure is represented by a *decision rule* $\delta = (\tau, d_\mu)$ which consists of a stopping time τ of the filtration $(\mathscr{F}_t^X)_{t\geq 0}$, $\mathscr{F}_t^X = \sigma(X_s; s \leq t)$, and an $\mathscr{F}_\tau^X$-measurable function d_μ taking values $1, \ldots, n$. The time τ corresponds to the moment of stopping the observation, and the value of d_μ to the hypothesis accepted.

With each decision rule (τ, d_μ) we associate the loss function $\mathscr{R}_X(\delta) = \mathsf{E}\big[c\tau + W(\mu, d_\mu)\big]$, where first component is the observation cost (proportional to the observation time) and second component is the (nonnegative) penalty for making a wrong decision. The problem consists in finding the decision rule $\delta^* = (\tau^*, d_\mu^*)$ with the minimal average loss, i. e. such that

$$\mathscr{R}_X(\delta^*) = \inf_\delta \mathscr{R}_X(\delta), \tag{2}$$

where the infimum is taken over all decision rules $\delta = (\tau, d)$ with $\mathsf{E}\tau < \infty$. In our talk we present a general way to tackle such kind of problems.

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3. First of all, we show that the process $M(X)$ defined by

$$M_t(X) = c_H \int_0^t s^{1/2-H}(t-s)^{1/2-H} dX_s$$

with

$$c_H = \left(\frac{\Gamma(3-2H)}{2H\Gamma(3/2-H)^3\Gamma(1/2+H)} \right)^{1/2}$$

is a *sufficient statistic* in (2). In the case $\mu = 0$ the process $M(X)$ is the well-known Molchan (or fundamental) martingale (see [3], [1]), but for $\mu \neq 0$ it appears to be a diffusion.

Next, we transform (2) to the similar problem but for *standard Brownian motion* and *nonlinear* cost of observation. Finally, we reduce our task to the *standard optimal stopping problem* for Brownian motion. To investigate it one can use methods from general theory (see, for example, [4], [5]).

Details and particular examples will be provided.

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Weak reflection principle and static hedging of barrier options

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The classical Reflection Principle is a technique that allows one to express the joint distribution of a Brownian motion and its running maximum through the distribution of the process itself. It relies on the specific symmetry and continuity properties of a Brownian motion and, therefore, cannot be directly applied to an arbitrary Markov process.

We show that, in fact, there exists a weak formulation of this method that allows us to recover the same results on the joint distribution of a Brownian motion and its running maximum. We call this method a Weak Reflection Principle and show that it can be extended to a large class of Markov processes, which do not posses any symmetry properties and are allowed to have jumps.

We demonstrate various applications of this technique in Finance, Computational Methods, Physics, and Biology. In particular, we show that the Weak Reflection Principle provides an exact solution to the problem of hedging Barrier options with a semi-static position in European type claims. Our method allows us to find such hedging strategies in the diffusion- and Lévy-based models. In addition, we show how it can be used to establish robust static hedging strategies that are model-independent. We illustrate the theory with numerical examples.

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Subdiffusive Ornstein-Uhlenbeck processes and applications to finance

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In this paper we analyse the subdiffusive structure of financial markets using the subdiffusive Ornstein-Uhlenbeck process as a model of asset prices. We use the subordinated Langevin equation approach to obtain model. The subordinator process is the inverse tempered stable distribution. Using that phenomena, the role of subordinator in Langevin equation is parellel to role played by Riemann-Liouville operator in fractional Fokker-Planck equation. We investigated the evolution of the probability density function of the subordinated Ornstein-Uhlenbeck process and we review the simulation of fractional Langevin equation. This model combines the mean-reverting behavior, long range dependence and trapping events properties of financial market. We applied the subordinated model to European call options for obtaining the fair option pricing formula.

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Exponential functionals of Lévy processes

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Consider the exponential functional

$$A_t = \int_0^t e^{\xi_s}\, ds, \tag{1}$$

and its terminal value $A_\infty = \lim_{t\to\infty} A_t$, where ξ_s is a Lévy process. The integral A_∞ naturally arises in a wide variety of financial applications as a stationary distribution of the generalized Ornstein-Uhlenbeck process,

$$V_t = e^{-\xi_t}\left(V_0 + \int_0^t e^{\xi_{s-}}\, ds\right),$$

see for instance papers about COGARCH (COntinious Generalized AutoRegressive Conditionally Heteroscedastic) model [3] and Paulsen's risk model [1]. A_∞ plays also a crucial role in studying the carousel systems [5], and self-similar fragmentations [2].

Denote the Lévy triplet of the process $-\xi_s$ by (c, σ, ν), i.e.

$$\xi_s = -\left(cs + \sigma W_s + \mathcal{T}_s\right), \tag{2}$$

where $\mathcal{T}_s$ is a pure jump process with Lévy measure ν. The aim of this study is to statistically estimate the Lévy triplet of ξ_s given by the observations of the integral A_∞. The algorithm presented below is based on the following recursive formula for the moments of A_∞, which is completely proved in [4]:

$$\mathbb{E}\left[A_\infty^{s-1}\right] = \frac{\phi(s)}{s}\,\mathbb{E}\left[A_\infty^{s}\right], \tag{3}$$

where $s \in \mathbb{C}$ is such that $0 < \Re(s) < \sup\left\{z \geq 0 : \mathbb{E}[e^{z\xi_1}] \leq 1\right\}$, and $\phi(s)$ is a Laplace exponent of the process ξ, i.e., $\phi(s) := -\log \mathbb{E}\left[e^{s\xi_1}\right]$. In particular case $\sigma = 0$, the Laplace exponent can be represented as

$$\phi(u + \imath v) = c\,(u + \imath v) - \int_0^{+\infty} e^{-\imath v x}\bar{\nu}(dx) + \int_0^{+\infty} \nu(dx), \qquad u, v \in \mathbb{R}, \tag{4}$$

where $\bar{\nu}(dx) = e^{-ux}\nu(dx)$. The formula (4) motivates the algorithm, which we describe below.

In what follows, we suppose that N observations $Y_1, ..., Y_N$ of the integral A_∞ are given.

1. Estimate the A_∞^s for $s = u + \imath v$, where u is fixed and v varies by $\widehat{\mathbb{E}}\left[A_\infty^s\right] = \sum_{i=1}^N Y_i^s / N$.

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2. Estimate $\phi(s)$ by $\hat{\phi}(s) = s\,\widehat{\mathbb{E}}\left[A_\infty^{s-1}\right]/\widehat{\mathbb{E}}\left[A_\infty^{s}\right]$. This estimate is based on (3).

3. Estimate c as a coefficient of the (asymptotical) regression problem:

$$\Im\hat{\phi}(s) = c\Im(s) - \Im\mathcal{F}_{\bar{\nu}}(-\Im(s)). \tag{5}$$

4. Estimate $a = \int_0^{+\infty} \nu(dx)$ by

$$\hat{a} := \text{mean}\left(\Re\hat{\phi}(s) - \hat{c}\Re(s)\right). \tag{6}$$

5. Estimate $\mathcal{F}_{\bar{\nu}}(v)$ in the points $\Im(s)$ by (4):

$$\hat{\mathcal{F}}_{\bar{\nu}}(-\Im(s)) = -\hat{\phi}(s) + \hat{c}s + \hat{a}. \tag{7}$$

6. Estimate ν by

$$\hat{\nu}(x) \quad = \quad \frac{1}{2\pi}e^{ux}\int_{\mathbb{R}} e^{\imath vx}\hat{\mathcal{F}}_{\bar{\nu}}(-v)\mathcal{K}(vh_n)dv, \tag{8}$$

   where $\mathcal{K}$ is a regularizing kernel supported on $[-1,1]$ and h_n is a sequence of bandwidths which tends to 0 as $n \to \infty$.

In this talk, we discuss some theoretical propeties of the proposed algorithm and provide examples.

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Pricing and hedging variance swaps on a swap rate

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We consider pricing and hedging of the generalized variance swap on a swap rate. It pays the weighted realized swap rate variance.

The variance swaps related market is very well established in equity derivatives. The foundations were laid independently by Neuberger [9] and Dupire [5] who came up with the model independent variance swap replication in terms of a dynamic trading strategy and a European payoff on the underlying stock or index. There have been many generalizations in terms of payoffs and also the underlying model: corridor variance swaps were considered in Carr and Lewis [3] and general weighted variance swaps were investigated in Lee [7]. The gamma variance swap, as in Lee [6] is the most common variance swap in equities. Variance swap pricing relies on the static replication techniques as in Carr and Madan [4].

In contrast to equities, the fixed income variance swaps have received very little attention so far. This is partially explained by the fixed income variance swaps market being relatively new. On the quantitative research side the same techniques used for equity derivatives can often be adapted to deal with the equivalent fixed income derivatives. This approach was adapted in Merener [8] who obtained the dynamic hedging strategy for the general weighted variance swap under certain yield curve assumption.

A number of questions have yet to be addressed. In particular, the restriction imposed on the yield curve is not compatible with the absence of arbitrage. It is also a wrong approximation in the degenerate case of a single period swap. In this case one should work with exact expressions since no approximations are needed. In general one needs to work under weaker assumptions in order to obtain arbitrage free hedging results.

In this research we suggest an alternative swap rate model to the classical approach. The latter is based on the underlying process approximations. In particular, we identify and approximate only the relevant *conditional expectation processes.* The exact conditional expectations being approximated as well as their number depend on each individual problem. The key feature of the model is that it exercises the degree of freedom between the conditional moments. Working under the minimal assumption allows us incorporating the seemingly conflicting functional relations into the model, in particular related to the absence of arbitrage.

We derive the absence of arbitrage constraint for such approximations. In particular, when the approximating function depends on the swap rate alone, we show that a certain conditional expectation has to be an affine function of the swap rate. On the other hand, absence of arbitrage usually does not imply functional forms for other conditional expectations and various functional relations between

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the underlying financial variables as well as market evidence may be incorporated into the model.

This approach is used to price and dynamically hedge the variance swap on a swap rate. We consider the generic payoff where realized variance is weighted by a function h. The hedging results are obtained for the general swap rate model approximation g. The dynamic strategy depends on the solution of the ODE. We also provide the solution that can be expressed as a double integral in terms of functions h and g. This result generalizes and also simplifies the result in Merener [8].

The pricing results we obtain are new. In particular we derive a different ODE for the pricing function. This ODE is simpler and in many cases can be solved explicitly. In order to obtain a variance swap price it remains to price a European payoff on the terminal swap rate. It is interesting that in general exact dynamic hedging is not possible for the swap rate variance swap. Hence a hedging strategy represents the average dynamic hedging. This yields that dynamic hedging strategy present value is not identically equal to that of a variance swap. We illustrate this by a numerical example.

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Hedging of barrier options via a general self-duality

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We study semi-static hedging of barrier options, and propose an extension of the known methods to cover the case of continuous stochastic volatility models when there is correlation between the price and the volatility process.

Semi-static refers to trading at most at inception and a finite number of stopping times like hitting times of barriers. The possibility of this hedge, however, requires classically a certain symmetry property of the asset price which has to remain invariant under the duality transformation. This leads naturally to the concept of self-duality which generalises the put-call symmetry, see [2] and more recently [3], [4]. To overcome the symmetry restriction, a certain power transformation has been proposed in the latter two papers which leads to the notion of quasi self-duality. While this works well in the context of exponential Lévy processes, see [6], it does not essentially change the picture for continuous stochastic volatility models. As has been shown in [5], a quasi self-dual price process in this setting is up to the costs of carry the stochastic exponential of a symmetric martingale. In particular, this would exclude any non-zero correlation between the volatility and the price process which is unrealistic.

We propose a different approach to deal with the correlated case: by a multiplicative decomposition, the price process is factorised into a self-dual and a remaining part. This latter part is used as a numeraire for a change of measure. Under this new measure called $\mathbb{R}$, replacing the risk-neutral measure $\mathbb{Q}$, the price process S is no longer a martingale but gets replaced by a modified price process D. We then show that always a generalisation of self-duality holds if one replaces in one side of the defining equation the measure $\mathbb{Q}$ by $\mathbb{R}$, and the process S by its modified form D, respectively.

This general self-duality allows to derive a semi-static hedge to barrier options as in the classical self-dual case. However, unlike in the latter case where one can combine two terms to one multiplied with the same indicator function, here one has to face two different indicator functions involving S respectively D. It turns out that one can estimate the difference $\log S - \log D$ by moments of a cumulative variance option, and can also express the relative entropy $H(\mathbb{R}, \mathbb{Q})$ in terms of those.

An alternative representation allows one to trade the barrier option at the hitting time for a time-dependent put option written on the modified price process, at zero cost. This can be either hedged dynamically, or else we propose the following: firstly, we use a put option written on the original price process as a semi-static hedge. This removes the time-dependency, since at the hitting time the price process equals the barrier level in our continuous stochastic volatility model. The

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difference to the put written on the modified price process can then be hedged dynamically, and we provide a representation of the resulting hedge portfolio.

Starting from the general representation, we then derive the dynamic hedging portfolios by Malliavin calculus, in particular the Clark-Ocone formula. In a stochastic volatility context, this necessarily involves higher greeks. Such an approach has been pioneered for European options in the Heston model in [1]. Here we adapt this approach to our specific situation, i.e. hedging of a time-dependent put option written on the modified price process under the measure $\mathbb{R}$, and generalise it to our general stochastic volatility framework.

This is joint work with Elisa Alós and Zhanyu Chen.

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On a generalized shadow price process in utility maximization problems under transaction costs

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1. We consider a discrete-time financial market model with bid-ask spread, where trader's terminal wealth X is given by

$$X_T(\gamma) = 1 + \sum_{t=0}^{T} (\underline{S}_t(\Delta\gamma_t)^- - \overline{S}_t(\Delta\gamma_t)^+), \quad \gamma_{-1} = 0,\ \gamma_T = 0; \quad \Delta\gamma_t := \gamma_t - \gamma_{t-1}.$$

Here $\underline{S}_t \le \overline{S}_t$ are the bid and ask prices of a risky asset (stock) and γ_t is the number of stock units in trader's portfolio at time moment t, $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$. Note that for the frictionless model ($S = \underline{S} = \overline{S}$) this formula shapes to the customary form: $1 + (\gamma \circ S)_T := 1 + \sum_{t=1}^{T} \gamma_{t-1}\Delta S_t$.

All processes are adapted to a filtration $(\mathcal{F}_t)_{t=-1}^{T}$, $\mathcal{F}_{-1} = \{\emptyset, \Omega\}$, $\mathcal{F}_T = \mathcal{F}$ on a probabilty space $(\Omega, \mathcal{F}, \mathsf{P})$. It is assumed that $\underline{S}_t, \overline{S}_t \in L^q(\mathcal{F}_t)$ for some $q \in [1, \infty]$ and $\gamma_t \in L^\infty(\mathcal{F}_t)$. We allow portfolio constraints of the form $(\gamma_t)_{t=0}^{T-1} \in \mathcal{Y}$, where $\mathcal{Y}$ is a convex subset of $\prod_{t=0}^{T-1} L^\infty(\mathcal{F}_t)$.

A functional $\Phi : L^q(\mathcal{F}_T) \mapsto [-\infty, \infty]$ is called *monotone* if $\Phi(X) \ge \Phi(Y)$ whenever $X \ge Y$, $X, Y \in L^q(\mathcal{F}_T)$ and *quasiconcave* if

$$\Phi(\alpha_1 X + \alpha_2 Y) \ge \min\{\Phi(X), \Phi(Y)\}$$

for all $X, Y \in L^q(\mathcal{F}_T)$, $\alpha_1 + \alpha_2 = 1$, $\alpha_i \ge 0$. We admit that trader's utility is represented by a monotone quasiconcave functional Φ.

Following [4] we call an adapted process $S \in [\underline{S}, \overline{S}]$ a *shadow price* if

$$\mu_S := \sup\{\Phi(1 + (\gamma \circ S)_T) : \gamma \in \mathcal{Y}\} = \lambda := \sup\{\Phi(X_T(\gamma)) : \gamma \in \mathcal{Y}\}.$$

Thus, it is impossible to outperform the optimal utility value λ related to the market with friction by trading at a frictionless shadow price. So, a shadow price can be interpreted as a least favorable frictionless price from trader's point of view.

2. The existence of a shadow price for the case of finite Ω (and a more traditional utility functional) was proved in [4]. In general a shadow price need not exist: see examples given in [1], [2], [5]. However, it was shown in [5] that the existence of a *generalized shadow price* process S^* is guaranteed under rather weak assumptions. This process corresponds to the *relaxed* utility functional.

To be precise, put

$$\Psi(S, \gamma) = \Phi(1 + (\gamma \circ S)_T)$$

and $\sigma_t = \sigma(L^q(\mathcal{F}_t), L^p(\mathcal{F}_t))$, where $1/p + 1/q = 1$. So, σ_t is the weak topology of L^q for $q \in [1, \infty)$ and the weak-star topology of L^∞. In any case the set

$$[\underline{S}_t, \overline{S}_t] = \{S_t \in L^q(\mathcal{F}_t) : \underline{S}_t \le S_t \le \overline{S}_t\}$$

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is σ_t-compact. Consider the vector space $\prod_{t=0}^T L^q(\mathcal{F}_t)$ with the product topology σ and denote by $\widehat{\Psi}(\cdot,\gamma)$ the σ-lower semicontinuous envelope (relaxation) of $\Psi(\cdot,\gamma)$ as a function on $[\underline{S},\overline{S}] = \prod_{t=0}^T [\underline{S}_t,\overline{S}_t]$:

$$\widehat{\Psi}(S,\gamma) = \sup_{V\in\mathcal{N}(S)} \inf_{S'\in V} \Psi(S',\gamma).$$

Here $\mathcal{N}(S)$ is the neighbourhood base at S of the topology σ, restricted to $[\underline{S},\overline{S}]$. As is known, $\widehat{\Psi}(\cdot,\gamma)$ is the largest σ-lower semicontinuous function majorized by $\Psi(\cdot,\gamma)$.

Consider the optimization problem for the relaxed functional $\widehat{\Psi}$:

$$\widehat{\mu}_S = \sup\{\widehat{\Psi}(S,\gamma) : \gamma \in \mathcal{Y}\}.$$

We call S^* a *generalized shadow price* if $\widehat{\mu}_{S^*} = \lambda$.

Theorem 1. *Let Φ be monotone and quasiconcave, $\underline{S}_t, \overline{S}_t \in L^q(\mathcal{F}_t)$, $t = 0,\dots,T$ for some $q \in [1,\infty]$. Then there exists a generalized shadow price $S^* \in [\underline{S},\overline{S}]$ and the following minimax relations hold true:*

$$\begin{aligned}\lambda = \sup_{\gamma\in\mathcal{Y}} \Phi(X_T(\gamma)) &= \sup_{\gamma\in\mathcal{Y}} \inf_{S\in[\underline{S},\overline{S}]} \widehat{\Psi}(S,\gamma) = \sup_{\gamma\in\mathcal{Y}} \inf_{S\in[\underline{S},\overline{S}]} \Psi(S,\gamma)\\ &= \inf_{S\in[\underline{S},\overline{S}]} \sup_{\gamma\in\mathcal{Y}} \widehat{\Psi}(S,\gamma) = \sup_{\gamma\in\mathcal{Y}} \widehat{\Psi}(S^*,\gamma) = \widehat{\mu}_{S^*}.\end{aligned}$$

Theorem 1 was deduced in [5] from the intersection theorem of [3] (Theorem 3). If there exists an optimal solution γ^* of the utility maximization problem for the market with friction, then a pair (γ^*, S^*), where S^* is a generalized shadow price, is exactly a saddle point of the relaxed utility function $\widehat{\Psi}(S,\gamma)$ [5] (Theorem 2.3).

If the original utility function $\Psi(S,\gamma)$ is already lower semicontinuous in S in an appropriate topology, Theorem 1 implies the existence of a shadow price. We present an example where $\Psi \neq \widehat{\Psi}$, but a generalized shadow price S^* is in fact a shadow price, and examples, where a generalized shadow price exists and a shadow price does not. In addition, we discuss the connection between shadow prices and duality theory.

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Portfolio selection and an analog of the Black-Scholes PDE in a Lévy-type market

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Recent developments on financial markets have revealed the limits of Brownian motion pricing models when they are applied to actual markets. Lévy processes, that admit jumps over time, have been found more useful for applications. Thus, we suggest a Lévy model based on Forward-Backward Stochastic Differential Equations (FBSDEs) for option pricing in a Lévy-type market. We show the existence and uniqueness of the solution to the following FBSDEs driven by a Lévy process L_t:

$$\begin{cases} P_t = p + \int_0^t f(s, P_s, W_s, Z_s)\, ds + \sum_{i=1}^{\infty} \int_0^t \sigma_i(s, P_{s-}, W_{s-})\, dH_s^{(i)}, \\ W_t = h(P_T) + \int_t^T g(s, P_s, W_s, Z_s)\, ds - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)}\, dH_s^{(i)}, \end{cases}$$

where P_t is the d-dimensional price process, W_t is the wealth process, Z_t is an $R^d \times \ell_2$-valued portfolio-related process, $H^{(i)}$'s are the orthonormalized Teugels martingales associated to the Lévy process L_t, and $[0, T]$ is an arbitrary time interval. Using our model, we describe the portfolio selection procedure in a Lévy-type market. Moreover, we present a Lévy analog of the Black-Scholes PDE as a partial integro-differential equation, and obtain its solution from the solution to the FBSDEs.

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Optimization of credit policy of bank and the government guarantees in a model of investment in a risky project

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There is a project that, in a certain period after investing (lag of fixed assets) can bring some (random) income flow. The project is risky, i.e. with probability q, $0 < q < 1$ it fails (remains unrealized) and investor defaults, and with probability $1 - q$ it starts acting.

It is assumed that the required investment I are credited by bank on certain terms (loan agreement). These terms include loan amount and duration, repayment of the loan principal, and interest. These elements make up the credit policy K of bank (in relation to given investment project).

If the project became operational, the repayment of loan and accrued interest on it starts after the lag, but in case of investor's default the loan is not returned. This in turn leads to higher interest rates.

In order to attract investment in risky projects, there is a mechanism of government guarantees. This means that if the project fails and investor does not return the credit, the bank receives compensation from the Government (reimbursement of credit) as a part θT of issued loan. This mechanism allows, in particular, to reduce the interest rate on the loan.

The tax system is represented in the model by coefficient γ of the tax burden, which is a share of tax payments in profits.

The aim of the investor is to solve an investment timing problem, i.e. on the base of the observed information on current market prices and the forecast of the future flow of profits from the project to choose the investment time in such a way that the expected net income value of the project (NPV), discounted to zero (base) time will be maximal.

This optimal time of investment τ^*, under certain assumptions about the process of profits π_t (see [1, 2]) is specified by a threshold that depends on the credit policy of the bank.

Knowing the dependence of investor behavior on credit terms, the bank chooses the credit policy, maximizing the expected discounted profits from the project, equal to the difference between the expected return on the loan (from the investor and the state) and the amount of the loan. This optimal credit policy is a function of the part of guaranteed reimbursement of credit.

Knowing the optimal credit policy of the bank and the corresponding optimal investor behavior as a function of the part of the reimbursement, the government determines the optimal part of a loan reimbursement so that appropriate budgetary effect will be maximal. We define the budgetary effect as the difference between the expected tax revenue from the project and the expected costs of the government on reimbursement of the bank loans (at the optimal behavior of the investor).

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So, an optimal investment time, optimal credit policy and optimal government guarantees can be viewed as Stackelberg equilibrium point in a three-player game.

Assuming that the profit flow is described by a geometric Brownian motion, we can obtain the explicit form of the optimal credit policy of the bank and the optimal part of the guaranteed reimbursement of credit.

In this case it is shown that the optimal part of the reimbursement is inversely proportional to the risk q and increases in the coefficient of tax burden. As for the dependence of the optimal part of the reimbursement on the volatility of the project, it is determined by the value of the tax burden. For small values of the tax burden (not exceeding a certain level), the optimal part decreases in volatility, and at higher values (above this level) increases. It is also shown that the optimal (from the bank's point of view) interest on the loan decreases linearly on the part of the reimbursement, and under the optimal part increases both in risk and volatility of the project, and decreases in tax burden.

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GARCH model with jumps augmented with news analytics data

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The work presented here tries to evaluate the impact of news on stock volatility through a small empirical study on augmented GARCH–Jumps models. While news analytics tools became more popular among investors as indicated in [1], there are not so much research works studying quantitative impact of news on stock volatility. It is worth to be mentioned the pioneering works [3] and [2]. In the paper of [3] firm-specific announcements were used as a proxy for information flows. It was shown that there exists a positive and significant impact of the arrival rate of the selected news variable on the conditional variance of stock returns on the Australian Stock Exchange in a GARCH framework. They split all their press releases into different categories according to their subject. In the second of the papers the author examines impact of news releases on *index* volatility. In the paper [4] was shown that the GARCH(1,1) model augmented with volume does remove GARCH and ARCH effects for the most of the FTSE100 companies, while the GARCH(1,1) model augmented with news intensity has difficulties in removing the impact of log return on volatility.

Based on empirical evidences for some of FTSE100 companies, it will be examined two GARCH models with jumps. First we consider the well-known GARCH model with jumps proposed in [5]. Then we introduced the GARCH-Jumps model augmented with news intensity and obtained some empirical results. The main assumption of the model is that jump intensity might change over time and that jump intensity depends linearly on the number of news. It is not clear whether news adds any value to a jump-GARCH model. However, the comparison of the values of log likelihood shows that the GARCH-Jumps model augmented with news intensity performs slightly better than "pure" GARCH or the GARCH model with Jumps.

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American put option valuation by means of Mellin transforms

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In this report we consider the Mellin transform derived on Black – Scholes formulae in year 2003 by Panini and Srivastav [1, 2]. The authors use Mellin transforms to derive at first an equation for the price of a European put on a single underlying stock and then extend to the Amercan put option[4, 5]. One of the techniques to solving Black – Scholes equations called Mellin transformation, and suggested by Panini [1, 2] for basket options was considered and was derived a new integral representations for the price and the free boundary of the American option in one dimension. We improve the numerical part of Panini's computations [1, 2, 3] by using the Newton's method for the free boundary condition [4, 5].

Consider now the American put option, the early exercise feature of this option gives rise to a free boundary problem. As far as we known, there exists no closed-form analytical expression for the value of American put and its free boundary. The Black–Scholes equation for the price of an American put $p(S,t)$ satisfies the nonhomogeneous equation

$$\frac{\partial p}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 p}{\partial S^2}+rS\frac{\partial p}{\partial S}-rp=f(S,t), \tag{1}$$

where t is current time $0<t<T$, $S(t)$ is asset price $0<S<\infty$, T is expiry date, r is the interest rate, σ is volatility of the market prices. The inhomogeneity is given by

$$f=f(S,t)=\left\{\begin{array}{ll} -r\cdot K, & \text{if} \quad 0<S\leq S^*(t) \\ 0 & \text{if} \quad S>S^*(t) \end{array}\right\}.$$

The final time condition is

$$P(S,T)=\theta(S)$$

and the free boundary is determined by

$$P(S^*,t)=K-S^*, \qquad \frac{\partial P}{\partial S}\mid_{S=S^*}=-1.$$

We take the Mellin transform and for the price $P(S,t)$ they get

$$P(S,t)=p(S,t)+\frac{rK}{2\pi i}\int_t^T\int_{c-i\infty}^{c+i\infty}S^{-v}\frac{(S^*(x))^v}{v}e^{\frac{1}{2}\sigma^2 h(v)(x-t)}\,dv\,dx. \tag{2}$$

Substituting $S=S^*(t)$ they get the following integral equation for the free boundary

$$K-S^*(t)=p(S^*(t),t)++\frac{rK}{2\pi i}\int_t^T\int_{c-i\infty}^{c+i\infty}\frac{1}{v}(\frac{S^*(t)}{S^*(x)})^{-v}e^{\frac{1}{2}\sigma^2 h(v)(x-t)}\,dv\,dx. \tag{3}$$

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Then we use the Newton's method for the calculation of free boundary $S^*(\tau)$. The Figure 1 shows the isolines of the function of the American put option as the smooth conjunction of the family of the strict vertical isolines with the curve linear isolines.

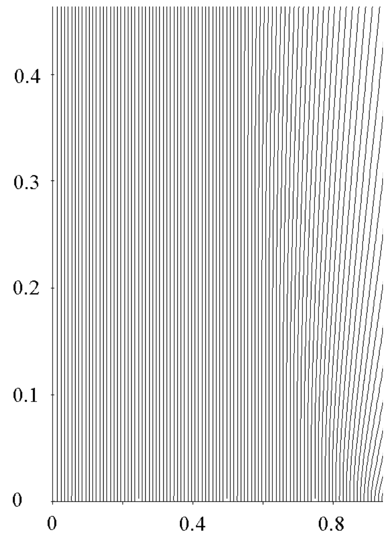

Figure 1: Isolines of the space for the function $p(s,\tau)$ with different choices for sigma option with $K = 45;\ \sigma = 0,3;\ T = 0,5833$

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Detection of trend changes in stock prices

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We consider a model of stock prices which initially increase but start to decrease after an unknown moment of time. The aim is to detect this moment and to sell the stock maximizing the gain. Applying methods of the theory of quickest changepoint detection, we show that the optimal detection rule consists in observing the posterior probability process (or, equivalently, the Shiryaev–Roberts statistics), and selling the stock as soon as this process exceeds a time-dependent threshold.

We apply the result to real market data and show that it provides good performance.

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Posters

About (B, S)-market model with stochastic switching of parameters

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Consider the (B, S)-market model:

$$dS_t = S_t[r(S_t)dt + \sigma(S_t)dW_t],$$
$$dB_t = r(S_t)B_t dt,$$

where $(S_t)_{t=0}^T$, $(B_t)_{t=0}^T$ – the value of assets, $(W_t)_{t=0}^T$ – Wiener process with respect to martingale measure P^*, $r(S_t)$ – interest rate, $\sigma(S_t)$ – volatility.

Suppose, that we have the barrier $M(t) = ce^{dt}$ with $c, d \equiv const$. The parameters of the model are switched in the stopping time

$$\tau = \inf\{0 \le t \le T : S_t = M_t\}.$$

Namely,

$$r(S_t) = \overline{r}_1 I_{\{0\le t<\tau\}} + \overline{r}_2 I_{\{\tau\le t\le T\}},$$
$$\sigma(S_t) = \overline{\sigma}_1 I_{\{0\le t<\tau\}} + \overline{\sigma}_2 I_{\{\tau\le t\le T\}},$$

where $\overline{\sigma}_1 > 0$, $\overline{\sigma}_2 > 0$.

The coefficients of the differential equation do not satisfy the standard conditions of existing and uniqueness of continuous solution, but the solution of the equation exists and is unique:

$$S_t = \begin{cases} S_0 \cdot \left(\exp\left(\overline{r}_1 - \dfrac{\overline{\sigma}_1^2}{2}\right) t + \overline{\sigma}_1 W_t\right),\ t \in [0, \tau], \\ M_\tau \cdot \left(\exp\left(\overline{r}_2 - \dfrac{\overline{\sigma}_2^2}{2}\right) (t-\tau) + \overline{\sigma}_2 W_{t-\tau}\right),\ t \in (\tau, T], \end{cases}$$

and

$$B_t = \begin{cases} B_0 e^{\overline{r}_1 t},\ t \in [0, \tau], \\ B_\tau e^{\overline{r}_2 (t-\tau)},\ t \in (\tau, T]. \end{cases}$$

Theorem 1 (see [2]). The fair price of the European call option $f_T = \max(S_T - K, 0)$ is $C = \mathsf{E}^* C(\tau)$ where $C(\tau)$ given by

$$C(\tau) = S_0 \Phi\left(\frac{-d(\tau) + \chi^2(\tau)}{\chi(\tau)}\right) - K\frac{B_0}{B_T}\Phi\left(\frac{-d(\tau)}{\chi(\tau)}\right)$$

with

$$d(\tau) = \ln\left(\frac{K}{S_0}\right) - \left(\overline{r}_1 - \frac{\overline{\sigma}_1^2}{2}\right)\tau - \left(\overline{r}_2 - \frac{\overline{\sigma}_2^2}{2}\right)(T-\tau),\ \chi(\tau) = \sqrt{\overline{\sigma}_1^2\tau + \overline{\sigma}_2^2(T-\tau)}.$$

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Here $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$ is the distribution function of the standard normal law and E^* denotes the expectation with respect to martingale measure P^*.

Theorem 2 (see [2]). Let $c > S_0$ and $\overline{r}_1 - \overline{\sigma}_1^2/2 - d > 0$. Then τ has the distribution:

$$p^*(x) = \begin{cases} g(x), & 0 \le x < T, \\ 1 - \displaystyle\int_0^T g(x)dx, & x = T, \\ 0, & x > T, \end{cases}$$

where

$$g(x) = \sqrt{\frac{b}{2\pi}} e^{ab} \frac{1}{x^{\frac{3}{2}}} \exp\left(-\frac{1}{2}\left(ax + \frac{b}{x}\right)\right)$$

and

$$\sqrt{a} = \frac{1}{\overline{\sigma}_1}\left(\overline{r}_1 - \frac{\overline{\sigma}_1^2}{2} - d\right), \quad \sqrt{b} = \frac{1}{\overline{\sigma}_1} \ln\left(\frac{c}{S_0}\right).$$

So,

$$\mathsf{E}^* C(\tau) = \int_0^T C(x) g(x) dx + C(T)\left(1 - \int_0^T g(x)dx\right).$$

Theorem 3 (see [1]). The following identity holds

$$P\left(\sup_{0 \le t \le T} (S_t - M_t) < 0\right) = \Phi\left(\frac{\ln\left(\frac{c}{S_0}\right) - \left(\overline{r}_1 - \frac{\overline{\sigma}_1^2}{2} - d\right)T}{\overline{\sigma}_1 \sqrt{T}}\right)$$
$$- \exp\left(\frac{2\ln\left(\frac{c}{S_0}\right)\left(\overline{r}_1 - \frac{\overline{\sigma}_1^2}{2} - d\right)}{\overline{\sigma}_1^2}\right) \Phi\left(\frac{\ln\left(\frac{S_0}{c}\right) - \left(\overline{r}_1 - \frac{\overline{\sigma}_1^2}{2} - d\right)T}{\overline{\sigma}_1 \sqrt{T}}\right).$$

At adequate selection of parameters of model the fair price will be less Black-Scholes price.

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Expected utility maximization in exponential Lévy models for logarithmic and power utility functions

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In this work we study the problem of utility maximization for a financial investor. We consider a model of financial market, which consists of one asset and has a finite maturity T. The price of the asset is modeled with a semimartingale S. The capital of the investor's portfolio has the form $X = x + H \cdot S$, where constant x denotes the initial wealth and H is a predictable S-integrable process, which specifies amount of the asset held in the portfolio. We assume that all capitals are strictly positive $\mathcal{X}(x) = \{X_t > 0 : X_0 = x\}$. The investor has a concave utility function U. His main aim is to maximize the expected terminal utility $EU(X_T)$, and we can denote the value function of this problem

$$u(x) = \sup_{X \in \mathcal{X}(x)} E[U(X_T)].$$

We can also consider the dual problem here. Let $V(y) = \sup_{x>0}(U(x) - xy),\ y > 0$. Then the value function of the dual problem is

$$v(y) = \inf_{Y \in \mathcal{Y}(y)} E\, V(Y_T).$$

Utility maximization and the dual problem were considered by Kramkov and Schachermayer in [3]. They indicated several properties for the solutions of these problems and connections between them. Some relations were also found for functions u and v. This research was made on general assumptions. In our work we show that in particular cases an explicit form and additional properties can be found for the solutions. To make this possible the process S is assumed to be a stochastic exponential of a Lévy process L, $\Delta L > -1$, which is completely determined by its triplet (b, c, ν). Besides, we consider only the following utility functions $U(x) = \ln x$ and $U(x) = x^p/p$. Kallsen [4] showed an explicit form for the solutions under several conditions, one of them de facto required the solution of the dual problem to be an EMM. In our previous works [1, 2] we solved this problem for the logarithmic utility for all possible triplets when L is not monotone, which is equivalent to the absence of arbitrage and can be stated in terms of the Lévy triplet, see [1]. No other restrictions were placed. The solution of the primary problem has the form $\mathcal{E}(\alpha L)$, where α is a constant. There are only three options for the solution of the dual problem and they can be characterized in terms of the triplet:

1. Y^* is the density process of an equivalent martingale measure.

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2. Y^* is a martingale, but not the density process of an equivalent σ-martingale measure.

3. Y^* is a supermartingale, but not a martingale.

Basically the restriction $u(x) < \infty$ is placed when solving the problem of utility maximization. But in the logarithmic case we can eliminate this restriction by considering the problem of finding the numéraire portfolio, which exists without the mentioned restriction. It coincides with the solution of primary problem X^* when the latter exists.

Now consider the case when $U(x) = x^p$, $0 < p < 1$. Here we cannot dismiss the condition mentioned above, so $EX_T^{*p} < \infty$. Let P be the initial measure. Then there exists a unique constant y^*, which determines the solution to the problem with logarithmic utility for measure Q, where Q is determined by Girsanov parameters $(\beta, Y) = (py^*, (1+y^*x)^p)$. $\mathcal{E}(y^*L)$ is the optimal portfolio for both logarithmic and power cases. Almost the same is true for the solutions Y_P^*, Y_Q^* of the dual problems regarding measures P and Q. Their ratio is equal to a constant:

$$Y_Q^*/Y_P^* = y_0,$$

where $y_0 = u'(1) = EX_T^{*p}$. From this fact we can easily derive that Y_P^* satisfies to one of the 3 options mentioned above and this can be stated in terms of the triplet. In such a way the solution to the power utility problem can be found via logarithmic case, applied to another measure Q. This method of solving differs from commonly used ones, which usually consider the logarithmic utility problem as a particular case of the utility with a power function, where p tends to 0.

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On optimal dividend payout in a factor diffusion model

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1. We consider a firm whose capital X is affected by an exogenous (macroeconomic) random factor Y, and assume that (X, Y) evolve according to the system of stochastic differential equations

$$\begin{aligned} dX_t &= (\mu(X_t, Y_t, \alpha_t) - c_t)dt + \sigma(X_t, Y_t, \alpha_t)dW_t, \\ dY_t &= \widehat{\mu}(Y_t)dt + \widehat{\sigma}(Y_t)\left(\rho dW_t + \sqrt{1-\rho^2}d\widehat{W}_t\right), \\ X_0 &= x, \quad Y_0 = y, \end{aligned}$$

where $(W, \widehat{W})$ are independent standard Wiener processes. The dividend-payment strategy $c_t \in [0, \overline{c}]$ and business plan strategy $\alpha_t \in \{1, \dots, n\}$ are progressively measurable with respect to the filtration, gererated by $(W, \widehat{W})$, and $\rho \in [0, 1]$ is a correlation factor. We assume that the stochastic control problem is in the standard form [9] (Chap. 3), that is, μ, σ, $\widehat{\mu}$, $\widehat{\sigma}$ satisfy uniform Lipschitz and linear growth conditions. The firm wants to maximize the expected total discounted dividends. The correspondent value function is given by

$$v(x, y) = \sup_{c,\alpha} \mathsf{E} \int_0^\tau e^{-\delta t} c_t dt,$$

where $\tau = \inf\{t \geq 0 : X_t = 0\}$ is the bankruptcy moment and $\delta > 0$ is a given rate of discount.

The optimal dividend policy problem in a diffusion model without external influence Y was first studied in [7, 4, 1], where it was established that a typical optimal policy is to pay maximal dividend rate when firm's capital X is above some critical level x^*, and to pay nothing when X is below x^*. The value function in the correspondent one-dimensional model was shown to be twice continuously differentiable. The influence of exogenous factors in the form of regime shifts, governed by a Markov chain, much more recently was examined in [8, 5]. Our aim is to study the case where an exogenous factor Y follows a diffusion process.

2. From the theory of stochastic optimal control it is known that, at least formally, the value function v satisfies the Hamilton-Jacobi-Bellman equation in the half-space:

$$\begin{aligned} &\inf_{\substack{c \in \{0, \overline{c}\}, \\ \alpha \in \{1, \dots, n\}}} \Big\{ \delta v - c - (\mu(x, y, \alpha) - c)v_x - \widehat{\mu}(y)v_y \\ &- \frac{1}{2}\left(\sigma^2(x, y, \alpha)v_{xx} + 2\rho\sigma(x, y, \alpha)\widehat{\sigma}(y)v_{xy} + \widehat{\sigma}^2(y)v_{yy}\right)\Big\} = 0, \quad x > 0, \qquad (1) \\ &v(0, y) = 0. \qquad (2) \end{aligned}$$

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More precisely, with the use of Bellman's optimality principle it can be shown that v is a *viscosity solution* of the above problem.

Denote by v^* the upper semicontinuous envelope of v. Under the assumption $\inf_{\alpha\in\{1,\dots,n\}} \sigma^2(0,y,\alpha) > 0$ with the use of methods of [2] (Proposition 1.1) we deduce that $v^*(0,y) = 0$. Based on this fact and following the argumentation of [9] (Theorem 6.21), we prove a comparison theorem, which leads to the following result.

Theorem 1. *Assume that* $\inf_{\alpha\in\{1,\dots,n\}} \sigma^2(0,y,\alpha) > 0$. *Then the value function* v *is a unique continuous viscosity solution of (1), (2). If, moreover, for any bounded domain* G *and any* $\alpha \in \{1,\dots,n\}$ *there exists a constant* $\theta > 0$ *such that*

$$\sigma^2(x,y,\alpha)\xi_1^2 + 2\rho\sigma(x,y,\alpha)\widehat{\sigma}(y)\xi_1\xi_2 + \widehat{\sigma}^2(y)\xi_2^2 \geq \theta(\xi_1^2+\xi_2^2), \quad (x,y) \in G, \ \xi \in \mathbb{R}^2,$$

then v *twice continuously differentiable in the open half-space* $x > 0$.

The second assertion of the theorem follows from the classical result of [3], as long as the uniqueness of a continuous viscosity solution is already proved. Note that the result of [3] cannot be applied directly, since it concerns a bounded domain.

To solve the problem numerically we use finite difference *degenerate elliptic* schemes, presented in [6].

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Pseudo binary differential evolution algorithm for cardinality constrained portfolio optimization

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Cardinality constrained portfolio selection is a very important topic for both practical portfolio management and academic research. The cardinality constraint put a limit on the number of assets in the portfolio. A huge number of algorithms and models (see, for example [1], [2], [3]) have been suggested to solve various representations of this problem.

Let N be the number of assets, K be the limit on the number of assets in the portfolio and let μ_i denote the expected return of asset i, $i = 1, \dots, N$. Let σ_{ij} be the covariance between the i and j assets returns, $i = 1, \dots, N$, $j = 1, \dots, N$; ρ be required level of expected return; $l_i \geq 0$ be the lower share limit price for i asset, $i = 1, \dots, N$, and $u_i \geq 0$ be the top share limit price for i asset, $i = 1, \dots, N$.

Let $0 \leq x_i \leq 1$ be the proportion of the total value of i-th asset, $i = 1, \dots, N$, that are invested, and δ_i is a variable, that equals to 1, if i-th asset exists in portfolio, 0 otherwise, $i = 1, \dots, N$.

Markowitz model [4] with discrete constraints on the value of the share invested in the asset and restrictions on the cardinality can be represented as follows:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} x_i x_j \to \min,$$

subject to

$$\sum_{i=1}^{N} \mu_i x_i = \rho, \quad \sum_{i=1}^{N} x_i = 1, \quad l_i \delta_i \leq x_i \leq u_i \delta_i, \ i = 1, \dots, N,$$

$$\sum_{i=1}^{N} \delta_i = K, \quad \delta_i \in \{0, 1\}, \ i = 1, \dots, N.$$

In this research we consider a metaheuristic approach using differential evolution algorithm [5], [6] for finding the efficient frontier of the Markowitz portfolio optimization problem with cardinality constraint.

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Swing options in the Black & Scholes model: a free-boundary approach

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Swing options are particular financial derivatives that may be described as American options with multiple exercises. They are widely traded in the energy markets and usually the option's underlying asset is the price of a given commodity. In mathematical terms the price of an option of this kind is described by the value function of an optimal stopping problem with multiple stopping times.

We assume that the dynamics of the price is a geometric Brownian motion and study the Swing put option on finite time horizon $T > 0$ with a strike $K > 0$ and two exercise rights. An important parameter of this problem is the so-called refraction period $\delta > 0$. It can be interpreted as the minimal period that the option's seller needs to deliver a new portion of asset. If the holder exercises its first right, then the second optimal exercise time is an optimal stopping time for a standard American put option. Therefore the double optimal stopping problem reduces to a single optimal stopping problem.

Using the local time-space calculus [5] we derive a closed form expression for the value function in terms of the optimal stopping boundaries for both exercise rights and show that the optimal stopping boundaries themselves can be characterised as the unique solution of nonlinear integral equations. These integral equations are then evaluated numerically.

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Sums of independent Poissonian subordinators and Ornstein–Uhlenbeck type processes in the sense of upstairs representation

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Poissonian Stochastic Index process (PSI - process ψ) is defined by a subordination of the random sequence (ξ_n), $n = 0, 1, \ldots$, to an independent Poissonian process $\Pi(s)$, $s \geq 0$, by the following way $\psi(s) = \xi_{\Pi(s)}$. We mainly focus on the case when (ξ_n) consists of i.i.d. random variables. We consider $(\psi_j(s))$, $j = 1, 2, \ldots$ the sequence of independent copies of the process ψ.

Basic Theorem. Let (ξ_n), $n = 0, 1, \ldots$, be i.i.d. rv's from the domain of attraction to a symmetrical α-stable law with the characteristic function $E\exp\{-it\xi_0\}$ $= \exp\{-|t|^\alpha\}$, $t \in R$, $\alpha \in (0, 2]$.

Then the following convergence of the finite dimensional distributions takes place as $N \to \infty$, $s \geq 0$,

$$\Psi_N^\alpha(s) = \frac{1}{N^{1/\alpha}} \sum_{j=1}^{N} \psi_j(s) \Rightarrow U^\alpha(s), \quad s \geq 0\,. \tag{1}$$

In the particular case of $\alpha = 2$ the right part in (1) is the standard Gaussian Ornstein-Uhlenbeck process with the viscosity coefficient $\lambda > 0$ which is explicitly equal to an intensity coefficient of the leading Poisson process Π. Constructive description of the limit of (1) for arbitrary $\alpha \in (0, 2]$ is based on the following embedding one-dimensional time s into the two-dimensional euclidian plane (R.L.Wolpert, M.S.Taqqu (2005)). Such kind embedding Wolpert and Taqqu call as "Upstairs Representation".

Let consider the Wiener-Chentsov random field on $B = [0, 1] \times (-\infty, +\infty) \ni (t, s)$: it is the symmetrical α-stable, $\alpha \in (0, 2]$, white noise $dZ^\alpha(t, s)$ with the scattering Lebesgue measure. Assume that $Z^\alpha(t, s)$ is normalized by such a way that for all $A \subset B : |A| = 1$,

$$\int_A dZ^\alpha(t, s) \overset{d}{=} L^\alpha(1)\,,$$

where $L^\alpha(v)$, $v \in [0, 1]$, is the symmetrical α-stable Levi process. For simplicity here we assume that the Poissonian intensity $\lambda = 1$. Since the upstairs representation is a case of the moving average representation, so we have to introduce the following 2-dimensional moving kernel, $s \geq 0$,

$$A_s = \Big\{ (v, r) : \; r \leq s; \; v \leq e^{-(s-r)}; \; (v, r) \in B \Big\}\,. \tag{2}$$

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Proposition. *Dynamics of the process U^α is described by the following moving average representation with a kernel of the "indicator type", $s \geq 0$,*

$$U^\alpha(s) = \int_{A_s} dZ^\alpha(v,u)\,. \tag{3}$$

Claims. We consider the following non-homogeneous case, when the leading Poissonian processes are given by the following distribution with a random intensity $\lambda > 0$, $\Pi(s) = N(\lambda s)$, where $N(s)$ is a standard Poissonian process of intensity 1. Let in Basic Theorem the corresponding Poissonian processes for (ψ_j), $j = 1, 2, \ldots$, have the distribution type of $\Pi(s) = N(\lambda s)$, $s \geq 0$, (λ_j) be i.i.d. positive intensities independent of the corresponding processes (N) and corresponding sequences (ξ). Let us distribution λ denote by ν. Then for the Gaussian case in (1) the covariance function for the limit exists and it is equal explicitly to the Laplace transform of the measure ν. For the non-Gaussian case the limit in (1) has a representation of the type (2), (3), when the moving kernel is the indicator function under the graphics of a function $g(s-r)$ (the argument $(s-r)$ corresponds notation(2)), where g is the Laplace transform of measure ν.

We apply our approach to processing the American Treasures financial data, kinds of zero-coupon bonds, and to the LIBOR rates.

Examples

0. The classical O-U process is a particular case of the measure ν which is degenerated at the point $\lambda > 0$.

1. A Simple curious example is as follows. Let the measure ν be the Γ distribution Γ_γ, the scale parameter $= 1$, and the shape parameter $\gamma > 0$. Then in Gaussian case the covariance function of the limit stationary process in (1) has the following long-memory property: $cov(s) = 1/(s+\mu)^\gamma$.

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Construction of a copula function from the joint distribution of Grubbs statistics

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1. Copula models widely use for modeling dependencies between random variables. Their applications include such fields as analysis of financial data and actuarial calculations. Copula function may be obtained by inversion from known joint distribution[1]. In this work we find the joint distribution of Grubbs test statistics and obtain a copula function from this bivariate distribution.

2. Let X_1, X_2, $\ldots$, X_{n-1}, X_n be a random sample from a normal $N(a,\sigma^2)$ distribution. Grubbs proposed the standardized maximum and minimum [2]:

$$T_n^{(1)} = (\max_{1\le i\le n}\{X_i\} - \overline{X})/S; \qquad T_n^{(1)} = (\overline{X} - \min_{1\le i\le n}\{X_i\})/S,$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2$.

It is known that $P(T_n^{(1)} < t) = P(T_{n,(1)} < t)$. Let $F_n^{(1)}(t)$ be the distribution function of $T_n^{(1)}$, then [3]:

$$F_n^{(1)}(t) = P(T_n^{(1)} < t) = \begin{cases} 0,\ t \le \frac{1}{\sqrt{n}},\ n \ge 2; \\ n \int\limits_{\frac{1}{\sqrt{n}}}^{t} F_{n-1}^{(1)}(g_n(x)) f_{T_n}(x)dx,\ \frac{1}{\sqrt{n}} < t \le \frac{n-1}{\sqrt{n}},\ n \ge 3; \\ 1,\ t > \frac{n-1}{\sqrt{n}}, n \ge 2; \end{cases}$$

$$\text{where } f_{T_n}(x) = \frac{1}{n-1}\sqrt{\frac{n}{\pi}}\Gamma\left(\frac{n-1}{2}\right)/\Gamma\left(\frac{n-2}{2}\right)\left(1 - \frac{n}{(n-1)^2}x^2\right)^{\frac{n-4}{2}}; \tag{1}$$

$$g_n(x) = \frac{n}{n-1}x\Bigg/\sqrt{\frac{n-1}{n-2}\left(1 - \frac{n}{(n-1)^2}x^2\right)}. \tag{2}$$

Let $\Lambda_n(t_1,t_2) = P(T_{n,(1)} < t_1, T_n^{(1)} < t_2)$ be the joint distribution function of Grubbs test statistics $T_n^{(1)}$ and $T_{n,(1)}$. The following theorem describes our main result.

Theorem. If X_1, X_2, $\ldots$, X_n is a random sample from $N(a,\sigma^2)$ distribution, then the joint distribution function of statistics $T_n^{(1)}$ and $T_{n,(1)}$ for the case $n=2$ is given by

$$\Lambda_2(t_1,t_2) = \begin{cases} 1, & (t_1,t_2) \in \Delta_2,\ \Delta_2 = [\frac{\sqrt{2}}{2} < t_1 < \infty; \frac{\sqrt{2}}{2} < t_2 < \infty]; \\ 0, & (t_1,t_2) \notin \Delta_2, \end{cases} \tag{3}$$

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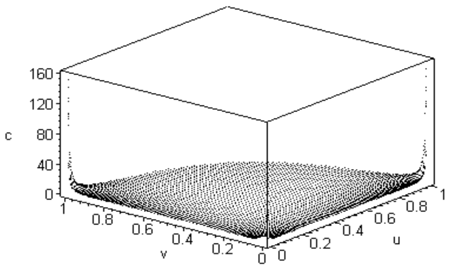

Figure 2: The graph of $c_{Gr}(u,v;5)$

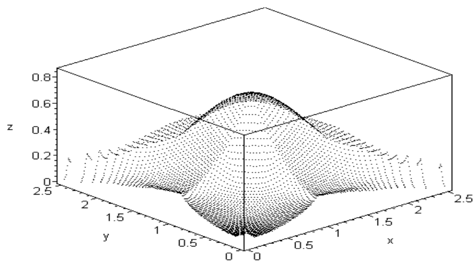

Figure 2: The graph of $h(x,y)$

and for the case $n > 2$

$$\Lambda_n(t_1,t_2) = \begin{cases} F_n^{(1)}(t_2), \ \ t_1 \geq \frac{n-1}{\sqrt{n}}; \\ F_n^{(1)}(t_1), \ \ t_2 \geq \frac{n-1}{\sqrt{n}}; \\ n \int\limits_{\frac{1}{\sqrt{n}}}^{t_2} \Lambda_{n-1}\left(\rho_n(t_1,-x), g_n(x)\right) f_{T_n}(x)dx, \ \ (t_1,t_2) \in \Delta_n; \\ 0, \ \ (t_1,t_2) \notin \Delta_n, \ t_1 < \frac{n-1}{\sqrt{n}}, \, t_2 < \frac{n-1}{\sqrt{n}}, \end{cases} \tag{4}$$

where $\rho_n(u,v) = (u + \frac{v}{n-1}) \Big/ \sqrt{\frac{n-1}{n-2}\left(1 - \frac{n}{(n-1)^2}v^2\right)}$, $|v| < \frac{n-1}{\sqrt{n}}$;
functions $g_n(x)$ and $f_{T_n}(x)$ may be calculated with using (2) and (1);
$\Delta_n = [1/\sqrt{n} < t_1 < (n-1)/\sqrt{n};\ 1/\sqrt{n} < t_2 < (n-1)/\sqrt{n}]$, if $n > 2$.

3. Grubbs's copula function may be obtained by inversion from bivariate distribution $\Lambda_n(t_1,t_2)$, i.e. $C_{Gr}(u,v;n) = \Lambda_n(t_1,t_2)$, where $u = F_n^{(1)}(t_1),\ v = F_n^{(1)}(t_2)$.

We carried out the computations of copula density $c_{Gr}(u,v;n) = \frac{\partial^2 C_{Gr}(u,v;n)}{\partial u \partial v}$ in the case $n = 5$. The figure 1 contains the result of computations.

As an example of modeling dependencies between random variables X and Y we discuss the case when X and Y have two-parameter Weibull distribution with marginals $F(x;\alpha_1;\beta_1)$ and $F(y;\alpha_2;\beta_2)$.

The function $H(x,y) = C_{Gr}(F(x;\alpha_1;\beta_1), F(y;\alpha_2;\beta_2);n)$ is a legitimate joint distribution function with marginals $F(x;\alpha_1;\beta_1)$ and $F(y;\alpha_2;\beta_2)$. We carried out the computations of density $h(x,y) = \frac{\partial^2 H(x,y)}{\partial x \partial y}$ for the case $n = 5$ with parameters $\alpha_1 = \alpha_2 = 1$ and $\beta_1 = \beta_2 = 2$. Figure 2 contains the result of calculations.

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On stochastic optimality in the portfolio tracking problem

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We consider the portfolio selection problem over an infinite time-horizon involving investor's time preference. The issue of stochastic optimality is also discussed.

Consider a self-financed portfolio consisting of n assets. The price vector of risky assets follows a multi–dimensional geometric Brownian motion with parameters $(\mu_t^{(i)}, \sigma_t^{(i)})$, $i=\overline{1,n-1}$ and r_t is the return on the risk–free (the n–th) asset. Denote by X_t the investor's total wealth at time t, starting with initial wealth $x>0$ and ignoring the transaction costs. We assume (see, e.g., [1]) that X_t, $t\geq 0$ is an R–valued stochastic process defined on a complete probability space $\{\Omega,\mathcal{F},\mathbf{P}\}$ by

$$dX_t = r_t X_t dt + B_t U_t dt + U_t' C_t dw_t\,, \quad X_0 = x\,, \tag{1}$$

where $B_t = \left(\mu_t^{(i)} - r_t,\, i=\overline{1,n-1}\right)$, $C_t = \left(diag(\sigma_t^{(i)}),\, i=\overline{1,n-1}\right)$;

$'$ represents the vector or matrix transpose; by $diag(\sigma_t^{(i)}), i=\overline{1,n-1}$ we denote the $(n-1)\times(n-1)$-diagonal matrix with diagonal elements $\sigma_t^{(i)}$, $i=\overline{1,n-1}$;

$\{w_t\}_{t=0}^{\infty}$ is an $n-1$-dimensional standard Wiener process; U_t, $t\geq 0$ is an admissible control, i.e. an $\mathcal{F}_t=\sigma\{w_s,\, s\leq t\}$–adapted $n-1$-dimensional process such that there exists a solution to (1). Note that $U_t^{(i)}$ defines the amount invested in the i–th risky asset. Let us denote by $\mathcal{U}$ the set of admissible controls. We don't impose any specific restrictions on U_t such as $\int\limits_0^{\infty} E\|U_t\|^2\,dt<\infty$, mean-square stability of the corresponding process X_t, etc., which are common for portfolio selection problems over the infinite-time horizon, see e.g. [2].

It is desired to determine the investment strategy U_t for tracking some investor-defined reference portfolio V_t^0. The reference portfolio is riskless and described by $dV_t^0=\rho_t V_t^0\,dt$, $V_0^0=x$. Notice that it's natural to assume $\rho_t>r_t$. The quadratic objective functional, see [3], measures the total loss from deviations of the actual portfolio and also the costs of portfolio control:

$$J_T(U^T) = \int_0^T f_t[(X_t - V_t^0)^2 + U_t' U_t]\,dt, \tag{2}$$

where $U^T=\{U_t\}_{t\leq T}$ is a restriction of $U\in\mathcal{U}$ to the finite horizon $[0,T]$, f_t is a discount function; it reflects the investor's time preference and satisfies the following

Assumption $\mathcal{D}$. The discount function $f_t>0$, $t\geq 0$ is non-increasing, its corresponding discount rate $\phi_t=-\dot{f}_t/f_t$ is bounded for $t\geq 0$ and $\int\limits_0^{\infty} f_t(V_t^0)^2\,dt<\infty$.

Consider the problem

$$\limsup_{T\to\infty} EJ_T(U) \to \inf_{U\in\mathcal{U}}\,. \tag{3}$$

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Assumption $\mathcal{P}$. Let $\tilde{A}_t = r_t - (1/2)\phi_t$. The triple $(\tilde{A}_t \;\; C_t \;\; B_t)$ is stabilizable.

The definitions of stabilizability for linear stochastic systems are given in [4]. Assuming $\mathcal{D}$ and $\mathcal{P}$, it can be shown that there exists a bounded positive semidefinite solution Π_t of generalized Riccati equation

$$\dot{\Pi}_t + 2\tilde{A}_t\Pi_t - \Pi_t B_t \tilde{R}_t^{-1} B_t' \Pi_t + 1 = 0\,, \tag{4}$$

where $\tilde{R}_t = I_{n-1} + C_t^2\Pi_t$, I_{n-1} is an $(n-1)\times(n-1)$ identity matrix.

Next, define

$$m_t = \int_t^{\infty} \Phi(s,t)\Pi_s l_s\, ds,$$

where $\Phi(s,t) = \exp\left(\int\limits_t^s (\tilde{A}_\tau - B_\tau \tilde{R}_\tau^{-1} B_\tau' \Pi_\tau)\, d\tau\right)$, $l_t = \sqrt{f_t}\,(r_t - \rho_t)\, V_t^0$. We proved the following

Theorem 1. Assume $\mathcal{D}$ and $\mathcal{P}$. Then the solution to problem (5) is given by

$$U_t^* = -\tilde{R}_t^{-1} B_t' (\Pi_t X_t^* - \Pi_t V_t^0 + m_t/\sqrt{f_t})\,, \tag{5}$$

the optimal portfolio process is governed by

$$dX_t^* = (r_t - B_t\tilde{R}_t^{-1}B_t'\Pi_t)X_t^* dt + B_t\tilde{R}_t^{-1}B_t'\,\tilde{l}_t\, dt + (\tilde{l}_t - \Pi_t X_t^*)B_t\tilde{R}_t^{-1}C_t dw_t\,, \tag{6}$$

where $\tilde{l}_t = \Pi_t V_t^0 - m_t/\sqrt{f_t}$ and $X_0^* = x$.

The issue of stochastic optimality is related to the comparison between $J_T(U^*)$ and $J_T(U)$ for $U \in \mathcal{U}$ in the almost sure (a.s.) sense. The next result is the following

Theorem 2. Let the conditions of Theorem 1 be satisfied. Then

a) for any $U \in \mathcal{U}$ there exists a finite (a.s.) random moment T_0 such that the inequality $J_T(U^*) - J_T(U) \le h_T$ holds (a.s.) for all $T > T_0$, where $h_T > 0$ is an arbitrary non-decreasing function such that $h_T \to \infty, T \to \infty$;

b) $J_T(U^*)$ converges a.s. to $J_\infty(U^*)$ as $T \to \infty$, where $J_\infty(U^*)$ is a random variable.

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